ABSTRACT

DERIVATION ON SOME SPECIAL TYPES OF RINGS AND THEIR PROPERTIES

By

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Let a ring $(R,+,\cdot)$ with a unity element, I is an ideal of R, and a multiplicatively closed set $A \subset R$ without zero elemen and without zero divisor elements. Using ring theory, a factor ring R/I and a division ring R_A if ring R is a commutative ring can be constructed. If $\{R_i\}_{i\in\Delta}$ is a family of rings, we can also construct a cartesian product ring $\prod_{i\in\Delta} R_i$. The concept of derivations on rings is motivated from the concept of derivatives in calculus by using Leibnitz's rules as a definition of derivations on rings. The derivation of the ring R is a mapping $d: R \to R$ where for each $a, b \in R$ satisfies the properties $d(a+b) = d(a) + d(b)$ and $d(a\cdot b) = d(a)\cdot b + a \cdot d(b)$. Suppose that $d: R \to R$ is a derivation in the ring R and I is the d-ideal of R, in this Final Project a derivation is constructed in the division ring of R_A, namely $d_A: R_A \to R_A$ where for each $\frac{r}{a} \in R_A$ we define $d_A\left(\frac{r}{a}\right) = \frac{d(r)\cdot a - r\cdot d(a)}{a^2}$ and the derivation in the factor ring R/I, namely $\bar{d}: R/I \to R/I$ where for each $r + I \in R/I$ is defined $\bar{d}(r+I) = d(r) + I$. If there is $\{d_i\}_{i\in\Delta}$ which is a family of derivations from the ring R_i for each $i \in \Delta$, then a derivation can be constructed in the cartesian product ring, namely $\prod_{i\in\Delta} d_i : \prod_{i\in\Delta} R_i \to \prod_{i\in\Delta} R_i$ where for each $(r_i)_{i\in\Delta} \in \prod_{i\in\Delta} R_i$ is defined $\prod_{i\in\Delta} d_i((r_i)_{i\in\Delta}) = (d_i(r_i))_{i\in\Delta}$.

Keyword: Division ring, factor ring, cartesian product ring, derivation on ring