

Locally stability analysis of the Phytoplankton-Nitrogen- Phosphate-Sediment dynamical system: A study case at Karimunjawa aquaculture system, Central Java

by Sapto Putro

Submission date: 09-Jul-2020 11:36AM (UTC+0700)

Submission ID: 1355276183

File name: ARTIKELC31-WIDOWATI.pdf (962.96K)

Word count: 2520

Character count: 12775

Locally stability analysis of the Phytoplankton-Nitrogen-Phosphate-Sediment dynamical system: A study case at Karimunjawa aquaculture system, Central Java

E Triyana^{1*}, Widowati², and S P Putro³

^{1,2}Department of Mathematics, Faculty of Science and Mathematics, Diponegoro University, Indonesia

³Department of Biology, Faculty of Science and Mathematics, Diponegoro University, Indonesia

*Corresponding author e-mail: triyanaeka03@gmail.com

Abstract. Disturbance of water environment due to organic enrichment by fish activities may result in the reduction of water quality and sediments. The relative importance of N and P limitation and released from organic sediment is still an open question. The aim of this paper is to analyze the locally asymptotic stable from a dynamical system model in the equilibrium of water ecosystem. Mathematical models are resultant of interaction among four main variables .i.e. nitrogen and phosphate concentration, phytoplankton abundance, and sediment in the water ecosystem at Menjangan Besar, Karimunjawa islands. The four variables are non-linear system differential equation that will form the dynamical system mathematical model. Local stability was determined by using Taylor series and Jacobian matrix. The system will be locally asymptotically stable if eigenvalues are negative. Numerical simulation was used to analyze the dynamic behavior of the system. From numerical simulation results base, it is concluded that equilibrium points. Because all eigenvalues of the Jacobian matrix were negative, the dynamic system model was locally asymptotically stable.

1. Introduction

Aquaculture can play an important role in meeting human demands in the form of protein-based foods. Indonesia must prepare itself to face the global challenges in the aquaculture industry or also known as fish farming [1]. Aquaculture is carried out in the marine environment and in underwater habitats. Improved fish farming produces a number of major problems for waters. wastewater contains compounds such as suspended solids, total nitrogen, and total phosphorus which can cause algal blooms, fish mortality, and increased sediment [2].

Nitrogen and phosphate are limiting nutrients that limit the growth of phytoplankton. Nitrogen is the basis for making proteins used by aquatic plants in the form of ammonia or nitrate, and plays a role in the formation of phytoplankton biomass as a producer of water, so that it becomes one of the parameters of fertility[3]. Phytoplankton has a direct response to nutrients through rapid changes in biomass, making it good to assess the ecological state of the waters[4]. In the phytoplankton cells, nitrogen and phosphate reflect the relative proportions needed by phytoplankton to grow [16]. A model of phytoplankton, zooplankton, and two complementary nutrients, nitrogen and phosphorus was system nutrients in governing system behavior and in turn in governing water quality control [15]

A complex sediment modeling has been developed to understand the processes that regulate pore water profiles and sediment-water exchange [12]. However, research on the limits of N and P released from sediments has not been done much, because of the complex interactions between the N and P cycles and factors external driving factors lead to dynamics in N or P restriction patterns and internal release that changes with time [13].

Analysis of the model using a linear analysis technique and found that the sinking of phytoplankton could affect the system. Local asymptotic stability around interior equilibrium is studied, which reveals that interior equilibrium loses its stability at some critical value of predation rate [5][6]

This paper aims is to modified the mathematical model from [7][8], to analyze the stability of the mathematical model. The researcher modified the model from the model [9][10].

2. Mathematical model

The dynamic model for aquaculture has been discussed which explains the influence of growth of phytoplankton and sedimentation with changes concentration of nitrogen and phosphate. The modified model changes phytoplankton, nitrogen, phosphate and sediment can be represented

$$\frac{dP}{dt} = \beta P - (\mu + \gamma)P, \quad (1)$$

$$\frac{dN}{dt} = q\Lambda + \omega\theta S - \gamma N - \alpha\beta P \frac{N}{N+F}, \quad (2)$$

$$\frac{dF}{dt} = (1-q)\Lambda + (1-\omega)\theta S - \gamma F - \alpha\beta P \frac{F}{N+F}, \quad (3)$$

$$\frac{dS}{dt} = \mu P - \theta S, \quad (4)$$

where $P(0) = P_0 \geq 0, N(0) = N_0 \geq 0, F(0) = F_0 \geq 0, S(0) = S_0 \geq 0$, with parameters β = phytoplankton growth rate (day⁻¹); μ = sedimentation rate of phytoplankton (day⁻¹); γ = water exchange rate (day⁻¹); q = proportion of N waste entering the water; Λ = total waste input (mg g⁻¹ day⁻¹); ω = proportion of remineralization rate of N in the sludge (with remainder remineralization Phosphate) (day⁻¹); θ = total remineralization sediment. Here P is phytoplankton concentration, N is nitrogen concentration, F is phosphate concentration, S is mass of N in sludge in pond water.

3. Locally stability analysis

Stability analysis is needed to analyze equilibrium points. Let (P^*, N^*, F^*, S^*) be the equilibrium point of the dynamical model nitrogen and phosphate concentration, phytoplankton, and sediment. The equilibrium point is obtained if satisfy $\frac{dP}{dt} = 0, \frac{dN}{dt} = 0, \frac{dF}{dt} = 0, \frac{dS}{dt} = 0$. We obtained the equilibrium point

$$(P^*, N^*, F^*, S^*) = \left(0, \frac{q\Lambda}{\gamma + \nu}, \frac{(1-q)\Lambda}{\gamma}, 0 \right).$$

Then the stability of non linear system at the equilibrium point (P^*, N^*, F^*, S^*) . First, we linearize the nonlinear system at the equilibrium point (P^*, N^*, F^*, S^*) using Taylor series as follows (1), (2), (3), (4)

$$\frac{d\bar{P}}{dt} = (\beta - \mu - \gamma) \bar{P}, \quad (5)$$

$$\frac{d\bar{N}}{dt} = \left(-\alpha\beta \frac{N^*}{N^* + F^*} \right) \bar{P} + \left(-\gamma - \nu - \frac{\alpha\beta P^*}{N^* + F^*} + \left(\alpha\beta P^* \frac{N^*}{(N^* + F^*)^2} \right) \right) \bar{N} + \left(\frac{\alpha\beta P^* N^*}{(N^* + F^*)^2} \right) \bar{F} + \theta \bar{S}, \quad (6)$$

$$\frac{d\bar{F}}{dt} = \left(-\alpha\beta \frac{F^*}{N^* + F^*} \right) \bar{P} + \left(\alpha\beta P^* \frac{F^*}{(N^* + F^*)^2} \right) \bar{N} + \left(-\gamma - \frac{\alpha\beta P^*}{N^* + F^*} + \left(\alpha\beta P^* \frac{F^*}{(N^* + F^*)^2} \right) \right) \bar{F} + (1-\omega)\theta \bar{S}, \quad (7)$$

$$\frac{d\bar{S}}{dt} = \mu\bar{P} - \theta\bar{S} \tag{8}$$

Furthermore, the linear differential equation system can be written in the form of a matrix equation as follow

$$\frac{d}{dt} \begin{bmatrix} \bar{P} \\ \bar{N} \\ \bar{F} \\ \bar{S} \end{bmatrix} = \begin{bmatrix} \frac{\partial G_1}{\partial P} & \frac{\partial G_1}{\partial N} & \frac{\partial G_1}{\partial F} & \frac{\partial G_1}{\partial S} \\ \frac{\partial G_2}{\partial P} & \frac{\partial G_2}{\partial N} & \frac{\partial G_2}{\partial F} & \frac{\partial G_2}{\partial S} \\ \frac{\partial G_3}{\partial P} & \frac{\partial G_3}{\partial N} & \frac{\partial G_3}{\partial F} & \frac{\partial G_3}{\partial S} \\ \frac{\partial G_4}{\partial P} & \frac{\partial G_4}{\partial N} & \frac{\partial G_4}{\partial F} & \frac{\partial G_4}{\partial S} \end{bmatrix} \begin{pmatrix} P^* \\ N^* \\ F^* \\ S^* \end{pmatrix} \begin{bmatrix} \bar{P} \\ \bar{N} \\ \bar{F} \\ \bar{S} \end{bmatrix},$$

where $\frac{d}{dt} \begin{bmatrix} \bar{P} \\ \bar{N} \\ \bar{F} \\ \bar{S} \end{bmatrix} = J \begin{pmatrix} P^* \\ N^* \\ F^* \\ S^* \end{pmatrix} \begin{bmatrix} \bar{P} \\ \bar{N} \\ \bar{F} \\ \bar{S} \end{bmatrix}$, with J is the Jacobian matrix at the equilibrium point (P^*, N^*, F^*, S^*)

$$J = \begin{bmatrix} \beta - \mu - \gamma & 0 & 0 & 0 \\ -\alpha\beta \frac{N^*}{N^* + F^*} - \gamma - \nu - \frac{\alpha\beta P^*}{N^* + F^*} + \left(\alpha\beta P^* \frac{N^*}{(N^* + F^*)^2} \right) & \frac{\alpha\beta P^* N^*}{(N^* + F^*)^2} & \omega\theta & 0 \\ -\alpha\beta \frac{F^*}{N^* + F^*} & \alpha\beta P^* \frac{N^*}{(N^* + F^*)^2} & -\gamma - \frac{\alpha\beta P^*}{N^* + F^*} + \left(\alpha\beta P^* \frac{F^*}{(N^* + F^*)^2} \right) & (1 - \omega)\theta \\ \mu & 0 & 0 & -\theta \end{bmatrix} \tag{9}$$

Furthermore, substitute the equilibrium point $(P^*, N^*, F^*, S^*) = \left(0, \frac{q\Lambda}{\gamma + \nu}, \frac{(1-q)\Lambda}{\gamma}, 0 \right)$ at Jacobian matrix

(9) so that it is obtained.

$$J = \begin{bmatrix} -(-\beta + \mu + \gamma) & 0 & 0 & 0 \\ -\alpha\beta \left(\frac{q\Lambda}{(\gamma + \nu) \left(\frac{q\Lambda}{(\gamma + \nu)} + \frac{(1-q)\Lambda}{\gamma} \right)} \right) & -\gamma - \nu & 0 & \omega\theta \\ -\alpha\beta \left(\frac{(1-q)\Lambda}{\frac{q\Lambda}{(\gamma + \nu)} + \frac{(1-q)\Lambda}{\gamma}} \right) & 0 & -\gamma & (1 - \omega)\theta \\ \mu & 0 & 0 & -\theta \end{bmatrix}$$

Then, we obtained $\det(\lambda I - J) = 0$

$$\det(\lambda I - J) = \begin{vmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{vmatrix} = \begin{vmatrix} -(-\beta + \mu + \gamma) & 0 & 0 & 0 \\ -\alpha\beta \left(\frac{q\Lambda}{(\gamma + \nu) \left(\frac{q\Lambda}{(\gamma + \nu)} + \frac{(1-q)\Lambda}{\gamma} \right)} \right) & -\gamma - \nu & 0 & \omega\theta \\ -\alpha\beta \left(\frac{(1-q)\Lambda}{\frac{q\Lambda}{(\gamma + \nu)} + \frac{(1-q)\Lambda}{\gamma}} \right) & 0 & -\gamma & (1 - \omega)\theta \\ \mu & 0 & 0 & -\theta \end{vmatrix} = 0$$

$$\det(\lambda I - J) = \begin{vmatrix} \lambda + (-\beta + \mu + \gamma) & 0 & 0 & 0 \\ \alpha\beta \left(\frac{q\Lambda}{(\gamma + \nu) \left(\frac{q\Lambda}{(\gamma + \nu)} + \frac{(1-q)\Lambda}{\gamma} \right)} \right) & \lambda + (\gamma - \nu) & 0 & -\omega\theta \\ \alpha\beta \left(\frac{(1-q)\Lambda}{\frac{q\Lambda}{(\gamma + \nu)} + \frac{(1-q)\Lambda}{\gamma}} \right) & 0 & \lambda + \gamma & -(1-\omega)\theta \\ -\mu & 0 & 0 & \lambda + \theta \end{vmatrix} = 0, \quad (10)$$

From the above characteristic equation, eigenvalues is obtained as follows

$$\lambda_1 = -(-\beta + \gamma + \mu)$$

$$\lambda_2 = -\gamma - \nu$$

$$\lambda_3 = -\gamma$$

$$\lambda_4 = -\theta$$

Therefore, the stability of the system of equation (1), (2), (3), (4) depends on the value of the characteristic roots. Based on the stability theory if all eigenvalues of the Jacobian matrix have a negative value, then the equilibrium point $(P^*, N^*, F^*, S^*) = \left(0, \frac{q\Lambda}{\gamma + \nu}, \frac{(1-q)\Lambda}{\gamma}, 0\right)$ on this system is locally

asymptotically stable

4. Results and discussions

The simulation will be used to analyze the dynamic system model, to check the optimal concentration of nitrogen and phosphate. Some numerical simulation by using data of nitrogen, phosphate, phytoplankton and sediment by [7] [8]. Research shows that there is a decrease in the concentration of nitrogen and phosphate, sediment, and phytoplankton in aquaculture. This is because the rest of the feed and excretion in the form of nitrogen and phosphate are utilized by marine biota. In order to validate the model equation (1) (2) (3) (4) some data parameter i.e $\Lambda = 3.98, \nu = 0.05, q = 0.9, \alpha = 0.54, \beta = 0.55, \mu = 0.85, \gamma = 0.068, \theta = 0.06$ Further equation (1), (2), (3), (4) is transformed to equation

$$\frac{dP}{dt} = -0.368P, \quad (11)$$

$$\frac{dN}{dt} = 3.582 + 0.0271S - 0.12N - 0.297P \frac{N}{N+F}, \quad (12)$$

$$\frac{dF}{dt} = 0.398 + 0.0008S - 0.068F - 0.297P \frac{F}{N+F}, \quad (13)$$

$$\frac{dS}{dt} = 0.85P - 0.00187S, \quad (14)$$

We obtained the equilibrium point is $(P^*, N^*, F^*, S^*) = (0.30, 3.559, 5.8529, 0)$. The mathematical model, data variable $P(0) = 0.952, N(0) = 0.68, F(0) = 0.56, S(0) = 0.67$ [10][11]. Substitution equilibrium points to Jacobian matrix obtained.

$$J = \begin{bmatrix} -0.368 & 0 & 0 & 0 \\ -0.1628 & -0.2229 & 0.125 & 0.00271 \\ -0.1341 & 0.1029 & -0.193 & 0.1029 \\ 0.85 & 0 & 0 & -0.0018 \end{bmatrix}$$

$$\det(\lambda I - J) = 0$$

$$\det(\lambda I - J) = \begin{vmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{vmatrix} - \begin{vmatrix} -0.368 & 0 & 0 & 0 \\ -0.1628 & -0.2229 & 0.125 & 0.00271 \\ -0.1341 & 0.1029 & -0.193 & 0.1029 \\ 0.85 & 0 & 0 & -0.0018 \end{vmatrix} = 0$$

$$\det(\lambda I - J) = \begin{vmatrix} \lambda + 0.368 & 0 & 0 & 0 \\ 0.1628 & \lambda + 0.2229 & -0.125 & -0.00271 \\ 0.1341 & -0.1029 & \lambda + 0.193 & -0.1029 \\ 0.85 & 0 & 0 & \lambda + 0.0018 \end{vmatrix}$$

$$= (\lambda + 0.368) \begin{vmatrix} \lambda + 0.2229 & -0.125 & -0.00271 \\ -0.1029 & \lambda + 0.193 & -0.1029 \\ 0 & 0 & \lambda + 0.0018 \end{vmatrix} - (0) \begin{vmatrix} 0.1628 & -0.125 & -0.00271 \\ 0.1341 & \lambda + 0.193 & -0.1029 \\ 0.85 & 0 & \lambda + 0.0018 \end{vmatrix}$$

$$+ (0) \begin{vmatrix} 0.1628 & \lambda + 0.2229 & -0.00271 \\ 0.1341 & -0.1029 & -0.1029 \\ 0.85 & 0 & \lambda + 0.0018 \end{vmatrix} - (0) \begin{vmatrix} 0.1628 & \lambda + 0.2229 & -0.125 \\ 0.1341 & -0.1029 & \lambda + 0.193 \\ 0.85 & 0 & 0 \end{vmatrix}$$

$$= (\lambda + 0.368)(0.00005428 + 0.0309\lambda + 0.4177\lambda^2 + \lambda^3)$$

$$= (\lambda + 0.368)(\lambda + 0.2229)(\lambda + 0.193)(\lambda + 0.0018)$$

From the above characteristic equation eigenvalues is obtained as follows $\lambda_1 = -0.368$ $\lambda_2 = -0.2229$ $\lambda_3 = -0.193$ $\lambda_4 = -0.0018$. Based on the theory of stability if the eigenvalues obtained are negative on that the equilibrium point $(P^*, N^*, F^*, S^*) = (0.30, 3559, 5.8529, 0)$ then this system is stable asymptotic. Therefore changes in the concentration of nitrogen and phosphate on sediments and phytoplankton in aquaculture form a stable system. Furthermore, to analyze the stability of the equilibrium point based on parameter values, a stability trajectory is drawn around the equilibrium point.

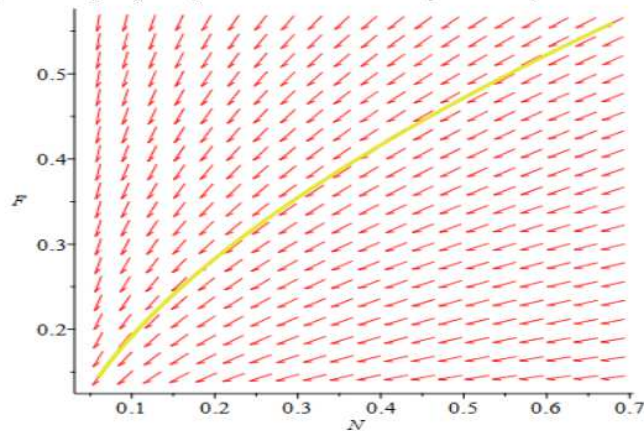


Figure 1. Trajectories stability around equilibrium points $(0, 30, 3559, 5.8529, 0)$ in (N, F) plane.

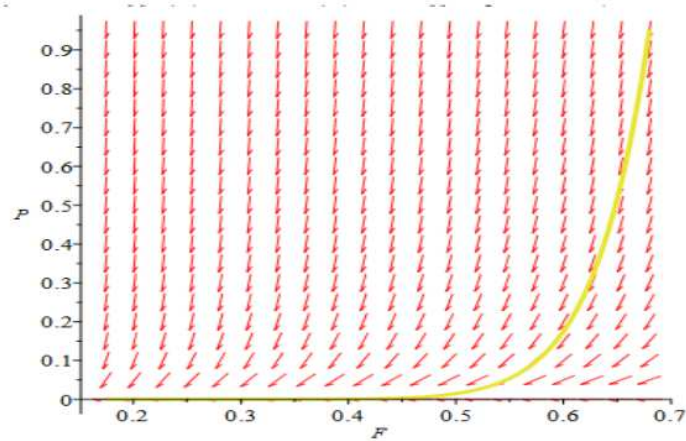


Figure 2. Trajectories stability around equilibrium point $(0,30.3559,5.8529,0)$ in (F, P) plane.

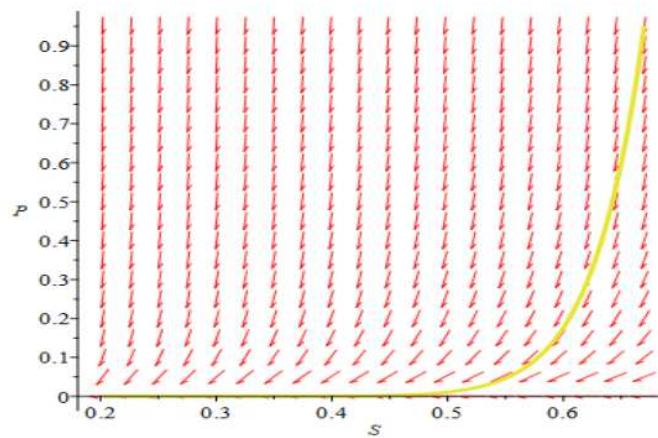


Figure 3. Trajectories stability around equilibrium points $(0,30.3559,5.8529,0)$ in the field (S, P) plane.

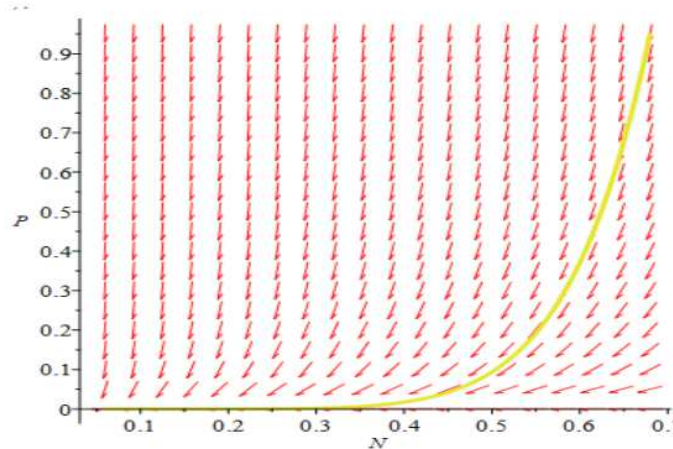


Figure 4 Trajectories stability around equilibrium points $(0,30.3559,5.8529,0)$ in the field (N, P) plane.

Based on Figure 1, Figure 2, Figure 3 and Figure 4 can see the direction of the arrow moving toward the equilibrium point $(P^*, N^*, F^*, S^*) = (0,30.3559,5.8529,0)$. This means that the equilibrium point for the given parameter is stable. The results of the dynamic system model showed that the concentrations of nitrogen and phosphate, phytoplankton and sediment decreased

5. Conclusion

It is concluded that model of nitrogen and phosphate concentration, phytoplankton abundance, and sediment is a non-linear dynamical system model. The numerical simulation concentration of nitrogen and phosphate are an important factor in the system dynamics. Eigenvalues are used to result analysis of the stability from the equilibrium point. Furthermore, analysis of locally stability of the dynamical system was based on the theory of stability. If the eigenvalues are negative, then the equilibrium point is locally asymptotically stable. From numerical simulation results base, it is concluded that equilibrium points and the eigenvalues are negative. Because all eigenvalues of the Jacobian matrix were negative, the dynamic system model was locally asymptotically stable. Therefore, the dynamical system phytoplankton population experience decline as a result of decreasing concentration N and P from sediment. This model is expected to be implemented to minimize the concentration of nitrogen and phosphorus properly, so that it can be utilized by marine biota.

Acknowledgments

The authors would like to thank Directorate General of Higher Education (Simlitabmas-DIKTI) for financially supporting this research project under the scheme of Excellent Research For National Strategic Grant year 2018 project. The study was supported by Laboratory of Computer Modelling, Mathematics Departement, Center of Marine Ecology and Biomonitoring for Sustainable Aquaculture (Ce-MEBSA), and Laboratory of Ecology and Biosystematics, Biology Departement, Faculty of Science and Mathematics, Diponegoro University.

References

- [1] Zhang Y, Bleeker A and Liu J 2015 *Environ. Res. Lett* **10** 045002
- [2] Turcios A E and Papenbrock J 2015 *Sustainability* **6** 836-856
- [3] Reynolds C S, Huszar V, Kruk C, Flores L N and Melo S 2002 *Journal of Plankton Research* **24(5)** 417-428.

- [4] Rast W, Holland M, Ryding S O 1989 *Eutrophication Management Framework For The Policy-Maker* Paris (FR): United Nations.
- [5] Wang Y, Zhao M, Dai C and Pan X 2014 *Hindawi Publishing Corporation Abstract and Applied Analysis* **451757**
- [6] Liu C and Liu P 2014 *Mathematical Problems in Engineering* **521917**
- [7] Burford M A and Lorenzen K 2004 *Aquaculture* **229** 129–145.
- [8] Deng Y, Zhao M, Yu H and Wang Y 2015 *Hindawi Publishing Corporation Discrete Dynamics in Nature and Society* **823026**
- [9] Widowati, Putro S P, and Silfiana 2018 *Journal of Physics*: **1025** 012088
- [10] Silfiana, Widowati, S P Putro and T Udjiani 2018 *Journal of Physics*: **983** 012122
- [11] Testa J M, Brady D C, Toro D M D and Boynton W R 2013 *Estuarine, Coastal and Shelf Science* **131** 245-263
- [12] Li Y, Liu Y, Zhao L, Hastings A and Gou H 2015 *Ecological Modelling* **313** 137–148
- [13] Widowati, Putro S P, Maan N and Sulpiani R 2019 *J. Phys.: Conf. Ser.* **1217** 012059
- [14] Granada L, Lopes S, Novais S C, Lemos M F L 2018 *Aquaculture* **495** 90–97
- [15] Antinios M N 1987 *Mathematical Biosciences* **84** 231-248
- [16] Olsen L M, Holmer M, and Olsen Y 2008 *Literature review with evaluated state of knowledge FHF project* 542014

a. Locally stability analysis of the Phytoplankton-Nitrogen-Phosphate-Sediment dynamical system: A study case at Karimunjawa aquaculture system, Central Java

ORIGINALITY REPORT

8%

SIMILARITY INDEX

2%

INTERNET SOURCES

7%

PUBLICATIONS

2%

STUDENT PAPERS

PRIMARY SOURCES

- 1** Burford, M.. "Modeling nitrogen dynamics in intensive shrimp ponds: the role of sediment remineralization", *Aquaculture*, 20040112
Publication 1%
- 2** M.N. Antonios. "A mathematical model of two-trophic-level aquatic systems with two complementary nutrients", *Mathematical Biosciences*, 1987
Publication 1%
- 3** www.hindawi.com
Internet Source 1%
- 4** projecteuclid.org
Internet Source 1%
- 5** Submitted to The University of Manchester
Student Paper 1%
- 6** Kai Yan, Zengwei Yuan, Stefanie Goldberg, Wei Gao, Anne Ostermann, Jianchu Xu, Fusuo Zhang, James Elser. "Phosphorus mitigation <1%

remains critical in water protection: A review and meta-analysis from one of China's most eutrophicated lakes", Science of The Total Environment, 2019

Publication

-
- | | | |
|----|---|-----|
| 7 | www.pyramexsafety.com Internet Source | <1% |
| 8 | Submitted to Central Queensland University Student Paper | <1% |
| 9 | TOSHIHIRO HAMACHI, WOLFGANG KRIEGER. "Embedding subshifts of finite type into the Fibonacci–Dyck shift", Ergodic Theory and Dynamical Systems, 2016 Publication | <1% |
| 10 | findresearcher.sdu.dk Internet Source | <1% |
| 11 | Lv, Xu Ying, Tian Wen Yao, and Ding Jiang Wang. "The Model of the Integrated Control of Plant Pests with Natural Enemy", Advanced Materials Research, 2013. Publication | <1% |
| 12 | P. Tamilalagan, S. Karthiga, P. Manivannan. "Dynamics of fractional order HIV infection model with antibody and cytotoxic T-lymphocyte immune responses", Journal of Computational and Applied Mathematics, 2021 Publication | <1% |

13

CHUNG, J.. "DYNAMIC ANALYSIS OF AN AUTOMATIC DYNAMIC BALANCER FOR ROTATING MECHANISMS", Journal of Sound and Vibration, 19991216

Publication

<1%

14

Yujue Wang, Dongyan Liu, Zhijun Dong, Baoping Di, Xuhong Shen. "Temporal and spatial distributions of nutrients under the influence of human activities in Sishili Bay, northern Yellow Sea of China", Marine Pollution Bulletin, 2012

Publication

<1%

15

Submitted to Flinders University

Student Paper

<1%

16

Submitted to Universiti Teknologi Malaysia

Student Paper

<1%

Exclude quotes Off

Exclude matches Off

Exclude bibliography Off