# Physics-based Simulation of Carrier Velocity in 2-Dimensional P-Type MOSFET

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### Abstract

The carrier velocity for 2-dimensional (2-D) p-type nanostructure was simulated in this paper. According to the energy band diagram, the effective mass (m\*) in the p-type silicon is mostly dominated by heavy hole because of the large gap between heavy hole and light *hole in* k = 0*. The carrier concentration calculation for* 2-D, based on the Fermi – Dirac statistic on the order of zero  $(\mathfrak{I}_0)$ , was applied to obtain the intrinsic velocity of carrier, in the term of thermal velocity  $v_{th}$ . The results for 2-D carrier velocity were modeled and simulated, and the comparison for degenerate and non-degenerate regime is presented for various temperature and concentration. It is revealed that the velocity is strongly dependent on concentration and becomes independent of temperature at high concentration.

## **1. Introduction**

In the progressive development of electronic devices nowadays, the deep understanding of the carrier behavior is vital for modeling and development of new appliances, ultimately when the device dimension reaches nanometer scale. It is widely known that the doping profiles of semiconductor material, its corresponding structure, energy band and ambient temperatures will influence the overall device performance, especially when the dimension is shrinking. Moreover, the carrier degeneracy at higher impurities needs to be specially considered for ultra small dimension, that it behaves differently from the non-degenerate one, which is commonly used in macroscopic/ bulk scale.

In nanoscale pMOS device, its channel region under the gate and gate oxide is very thin (in the order of nm), thus the carriers are confined in this direction. On the contrary, the carriers can travel easily in other directions, as its channel length and also channel width are larger than the de Broglie wavelength  $\lambda_D$  (~10 nm at room temperature) [1]. Therefore, this device can be considered as a 2-Dimensional device.

Previous models were based on charge calculations for nanoscale transistor modeling. But their work was based on the Maxwell Boltzman approximation (nondegenerate regime) [2-4]; on the other hand the nanoscale devices also operate in degenerate regime. Some literatures discuss the carrier statistic and velocity in 2-D system, but the specific discussion on the p-type device in nanoscale is not sufficiently deliberated, especially with the view on the intrinsic velocity [5].

In this research we improved velocity approach in nanoscale transistor modeling for both degenerate and nondegenerate regime. This paper is an extended work of Arora et.al[6] in determining the velocity of carrier in nanoscale device for n-type device. The following passages will discuss about the analytical modeling of the carrier velocity in 2-dimensional PMOS device, derived using physical approach, as well as the parametrical simulation in the case of degeneracy and non-degeneracy at various ambient temperatures and concentrations.

## 2. Carrier statistic model

In 2D device, one side of the device has its length confined to less than the de Broglie wavelength  $\lambda_D$ , but the other two directions are much larger than that number (see Fig. 1, with the confinement is assumed in

z-axis direction). Therefore energy spectrum can be modeled as follows:



**Fig 1.** Illustration of 2-D system, confined in the z-axis direction ( $L_z < \lambda_D \approx 10$ nm,  $L_{x,y} >> \lambda_D$ )

$$E = E_{v0} - \frac{\hbar}{2.m^*} \left( k_x^2 + k_y^2 \right) - n^2 \frac{\pi^2 \hbar^2}{2m^* L_z^2}$$
(1)

where  $E_{vo}$  is the energy at valence band,  $\hbar = h/2\pi$  is the Planck's constant,  $m^*$  is the effective mass of hole, n is the quantum number,  $k_x$  and  $k_y$  are the wave number for x and y direction, respectively, at quantum number of n as noted by  $k_{(x,y)n} = n\pi/L_{(x,y)}$ , and  $L_z$  is the length of the confinement in z-axis. Effectively, the hole is concentrated in the lowest subband at first quantum state, n=1.

The valence band consists of heavy hole (hh) and light hole (lh) bands. According to the energy band diagram and the occupation ratio between both bands, the p-type carrier is mostly dominated by heavy hole, therefore hole transport in p-channel MOSFETs is controlled by the properties of hh band, with the overall effective mass  $m^* = 0.48 m_0 [7, 8]$ .

The probability of hole occupation in any state of energy  $E_k$  is given by the complementary of Fermi-Dirac distribution function,

$$1 - f(E_k) = \frac{1}{1 + e^{\frac{E_F - E_k}{k_B T}}}$$
(2a)

where  $E_{Fl}$  is the Fermi energy at which the probability of occupation is half and T is the ambient temperature. In the non-degeneracy case, this complementary Fermi-Dirac distribution function will have the large value of  $E_F - E_k$ , thus eq. (2) is essentially turned to simpler yet valid formulation, the broadly popular complementary Maxwell-Boltzmann approximation for p-type devices:

$$1 - f(E_k) = e^{-\left(\frac{E_F - E_k}{k_B T}\right)}$$
(2b)

On the other hand, the shrinking of recent electronic devices into nanoscale bring about the need of higher carrier concentration that the use of degeneracy model is a necessity now. Moreover, the extensive derivation to obtain density of states D(E) in 2-D p-type MOSFET can be found in [9] while the final formula is shown as follows:

$$D(E) = \frac{1}{A} \frac{dN}{dE} = -\frac{1}{2\pi} \left( \frac{2m^*}{\hbar^2} \right)$$
(3)

# 3. Carrier velocity formulation

Let us take a look at the kinetic energy and its relation with the valence band  $E_{v0}$  and the energy state E of a hole:

$$E_{v0} = E + \frac{1}{2}m^*v^2 \tag{4}$$

The arbitrary velocity of a hole can be directly derived as:

$$v = \sqrt{\frac{2(E_{v0} - E)}{m^*}}$$
(5)

Take a note that the thermal velocity of hole is widely known as

$$v_{th} = \sqrt{\frac{2k_BT}{m_{hh}^*}} \tag{6}$$

In the presence of high electric field  $\varepsilon$ , the carrier will eventually move in homogeneous direction, parallel to the electric field, while in the absence of electric field, the average velocity of all carrier will be zero. The average drift velocity of carrier in the presence of  $\varepsilon$ , or we call it "intrinsic velocity" [6] can be calculated using formula below:

$$v_{i} = \frac{\int\limits_{E_{bottom}}^{E_{V}} |v| D(E) [1 - f(E)] dE}{\int\limits_{E_{bottom}}^{E_{V}} D(E) [1 - f(E)] dE}$$
(7)

In order to simplify the integration in Eq. (7), we introduce two terms,

$$x = (E_v - E_k)/k_B T \tag{8a}$$

$$\eta_v = (E_v - E_F) / k_B T \tag{8b}$$

Another important tool that needs to be introduced here is the Fermi integral  $\mathfrak{T}_{i}(\eta)$ , which is defined as:

$$\mathfrak{I}_{j}(\eta) = \frac{1}{\Gamma(j+1)} \int_{0}^{\infty} \frac{x^{j}}{e^{(x-\eta)} + 1} dx \qquad (9)$$

where  $\Gamma(j)$  is the Gamma function of *j*. The Fermi integral in 0-th order  $\mathfrak{T}_0(\eta)$  is shown in Fig. 2.



Fig. 2 The Fermi integral in  $0^{th}$  order as a function of  $\eta$ 

By combining Eqs. (2)-(6)) and substituting them into Eq. (7), added with some manipulations using Eqs. (8a), (8b) and (9), the intrinsic velocity  $v_i$  could be calculated and solved numerically as follow:

$$v_i = v_{th} \frac{\Gamma(3/2).\mathfrak{I}_{1/2}(\eta_F)}{\Gamma(1).\mathfrak{I}_0(\eta_F)}$$
(10a)

$$v_i = v_{th} \frac{\sqrt{\pi}}{2} \frac{\mathfrak{T}_{1/2}(\eta_F)}{\mathfrak{T}_0(\eta_F)}$$
(10b)

#### 4. Simulation results and discussions

According to equation (10b), we have the model of carrier velocity in 2-dimensional p-type device. Let us take a closer look on the temperature effect and also the influence of carrier concentration on the carrier velocity, both for degeneracy and non-degeneracy cases. By recalling eq. (2b) for non-degeneracy case, the Fermi integral of 2D devices in j-th order can be simplified to:

$$\mathfrak{I}_{j}(\eta_{v}) = e^{\eta_{v}} \tag{11}$$

Substituting eq. (11) into (10b), therefore the intrinsic velocity of non-degeneracy regime can be converted to:

$$v_i = \frac{\sqrt{\pi}}{2} v_{ih} \tag{12}$$

This formula is also the limit of intrinsic velocity of low concentration device. To validate the model and simplification presented by eqs. (10)-(12), the carrier velocity of 2-D p-type MOS is numerically simulated. Fig. 3 clearly shows that the intrinsic velocity of carrier is strongly dependent to temperature  $(T^{1/2})$  for lower concentration, as expected from eq. (13) and eventually reaches the limit of non-degenerate model, for low carrier concentration. It also reveals the lesser influence of temperature on the intrinsic velocity when the carrier concentration is higher, which tends to go to degenerate regime.

For strongly degenerate case, the Fermi integral is approximately transformed to:

$$\mathfrak{I}_{j}(\eta) \approx \frac{1}{\Gamma(j+1)} \frac{\eta^{j+1}}{j+1} = \frac{\eta^{j+1}}{\Gamma(j+2)} \quad (13)$$

Later, the corresponding intrinsic velocity eventually becomes:

$$v_i = \frac{2}{3} v_{th} . \eta^{1/2}$$
 (14)

Equating further using (6) and (8a), this result will give us further impression that the intrinsic velocity will behave independently over temperature at degenerate regime. Now let us consider that the carrier statistic of p-type is (from [9]) :

$$p = \frac{m^* k_B T}{\pi \hbar^2} \cdot \mathfrak{I}_0(\eta_v) = N_V \cdot \mathfrak{I}_0(\eta_v) \qquad (15)$$

Taking the notation of  $\mathfrak{T}_0(\eta)$  from (15) and inserting it back to eq. (10b), we find that the intrinsic velocity is correlated to the carrier concentration, as an alternative notion to eq. (14):



Fig. 3 Intrinsic velocity v<sub>i</sub> versus temperature for several carrier concentration of p-type MOS



**Fig. 4** The role of carrier concentration in the intrinsic velocity, with the degeneracy case shown as limiting value

$$v_i = v_{th} \frac{\sqrt{\pi}}{2} \mathfrak{I}_{1/2}(\eta_F) \frac{p}{N_V}$$
(16)

Manipulating back eqs. (8), (14) and (16), we will finally have:

$$v_i = \frac{2}{3} \sqrt{\frac{2(E_V - E_F)}{m^*}}$$
(17)

$$v_i = \frac{2\hbar}{3m^*} \sqrt{2\pi p} \tag{18}$$

The latest equation clearly suggests that the intrinsic velocity is directly related to the carrier concentration  $p^{1/2}$  for higher carrier concentration, in the strong degenerate regime. The simulation of (Fig. 4) clearly supports the argument, in addition with the independence of  $v_i$  from temperature variation for higher concentration, as evident from the indifference of graphs of various temperature at concentration higher than  $10^{18}$  cm<sup>-2</sup>, which eventually meet with the degenerate intrinsic velocity. The figure also suggests that the intrinsic velocity in degenerate regime is the velocity limit at lowest temperature possible.

The saturation velocity can be predicted from this simulation. Following the nature of maximum concentration possible in silicon semiconductor, the ultimate velocity that can be acquired in p-type device would be around the order of  $10^5$  m/s, as if we follow the trend from simulation. Moreover, the tendency of velocity dependence on  $p^{1/2}$  can provide further explanation on the expected bending of velocity curve in higher concentration.

#### 5. Conclusion

We have presented the physics-based carrier velocity model of 2-Dimensional pMOS silicon device. By considering quantum state of 2D device, the carrier statistic was modeled and the intrinsic velocity was simulated as well. The intrinsic velocity in the presence of external electric field is closely related to thermal velocity in the case of non-degenerate regime, and strongly dependent on the ambient temperature in  $T^{1/2}$  form, while in the case of strong degeneracy, the carrier concentration is influential in the velocity, proportional to  $p^{1/2}$ . The simulation also reveals the limit for intrinsic velocity, either in temperature variation or in carrier concentration, and it is expected that the ultimate velocity will be around that value.

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#### 7. References

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