Numerical Solution of 2-D Pollutant Distribution Model Based on Advection-Diffusion Mechanism in Waste Stabilization Ponds

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Numerical Solution of 2-D Pollutant Distribution Model Based on Advection-Diffusion Mechanism in Waste Stabilization Ponds

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This paper presents solution of Biochemical Oxygen Demand (BOD) concentration distribution model in waste stabilization ponds on the surface (2-D) based on the advection-diffusion mechanism of some cases. This model is represented in second-order partial differential equations. The purpose of this paper is investigate the BOD distribution design in waste stabilization ponds of some cases. One of the numerical methods that can be used to solve the model in the form of partial differential equations are finite difference method. Finite difference method consists of several schemes. The scheme is expected to provide good results is a Forward Time Central Space scheme. The initial step in this method is the process of discretization using finite difference forward to derivative of time and finite difference central to derivative of space. The discrete equation will be substituted into the partial differential equations. The results show that a decrease in the concentration of BOD every time caused by the movement of pollutants (advection and diffusion). It can be seen from the numerical simulation of all cases.

Keywords: Advection-Diffusion, Biochemical Oxygen Demand, Finite Difference, Forward Time Central Space, Waste Stabilization Ponds.

1. INTRODUCTION

Wastewater, both domestic waste and industrial wastewater is still a serious problem. The process of wastewater treatment in urban areas are becoming an indispensable requirement. In the wastewater treatment process, particularly those containing a lot of pollutant organic compounds, the technology used mostly using the activity of micro-organisms that will outline the pollutant organic compounds, commonly known as biological treatment processes. Biological treatment processes commonly used in Indonesia is the pool system or better known as waste stabilization ponds.

Waste stabilization ponds are pools of large land and shallow, where the wastewater put into the pool with a residence enough long time to enable the natural biological purification, according to the degree of processing are determined.[1] In waste stabilization ponds, wind energy and gravity provide the movement of wastewater that led to the mass transport process. The context of the movement in the waste stabilization pond system can be divided into two general mechanisms, namely advection and diffusion.

Advection is a mechanism that moves a substance or material from one position to another in space, or in other words referring to mass displacement by the fluid velocity. While diffusion related to the mass movement due to the random motion of water molecules.[2] Advection-diffusion is two mechanisms which absolute occur in waters. However, the design of movement of this mechanisms is difficult to detect. Moreover, advection-diffusion process is always associated with the boundary conditions and the amount of the initial concentration of pollutants. As one of the ways that can be used to determine the advection-diffusion design is to do modelling.

With the pollutant gradient horizontally and vertically in columns pond, the phenomenon of solute transport by advection and diffusion mechanisms can be described in 2-D models. Mathematical model of 2-D advection-diffusion is represented by using partial differential equations based on the phenomenon of physics, chemistry and biology.

The various studies have been conducted on waste stabilization ponds and associated with the advection-diffusion mechanism along with the solution of both analytically and numerically, such as builds models of CFD (Computation Fluid Dynamics) that describe the hydrodynamics of a full-scale waste stabilization ponds,[3] then presents a mathematical model derived from physical and biochemical phenomena in biological treatment process (facultative waste stabilization ponds).[4] Then, there are research which solved the advection-dispersion equations using finite difference method FTCS (Forward Time Central Space).[5] Then also,

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the advection diffusion equation 2-D solved by using the finite difference method Duffort Frankel.[6,7] Furthermore, the advection diffusion equation linear one-dimensional solved by using separation of variables and numerically using the finite element method.[8] Based on research that has been done, it is known that the finite difference method based on FTCS scheme is a better than finite difference method based on Duffort Frankel scheme and other methods.

Wherefore the wastewater treatment using BOD (Biochemical Oxygen Demand) as indicator to determine the quality of wastewater, so we conducted research on the numerical solution of the 2-D BOD concentration distribution model on waste stabilization ponds by advection-diffusion mechanism based on some cases boundary and initial conditions. In this case, the BOD concentration on the surface to be studied their changes through time. Changes in 2-D BOD concentration through time can be represented in the form of partial differential equations. Therefore, the research conducted in the stabilization ponds which are domain of simple field with a structured grid discretization process, the numerical method used for the solving of the model are finite difference method FTCS.

2. EXPERIMENTAL DETAILS

The concentration of pollutants in waste stabilization ponds move in the *one* direction with velocity u and diffuses in the direction of x and y with respectively diffusion coefficients D_{mx} and D_{my} are described in the following equation:[9]

$$\frac{\partial C}{\partial t} = -\frac{\partial (uC)}{\partial x} + D_{mx}\frac{\partial^2 C}{\partial x^2} + D_{my}\frac{\partial^2 C}{\partial y^2} \tag{1}$$

Second-order linear partial differential equations in this study resolved with finite difference methods based on FTCS scheme. Discretization an FTCS scheme shown in Figure 1.

Finite difference method selected for the research carried out on a simple domain with discretization structured process, namely waste stabilization ponds shaped bar. Then, based on the study of literature, the finite difference method which gives good results is finite difference method based on FTCS scheme.

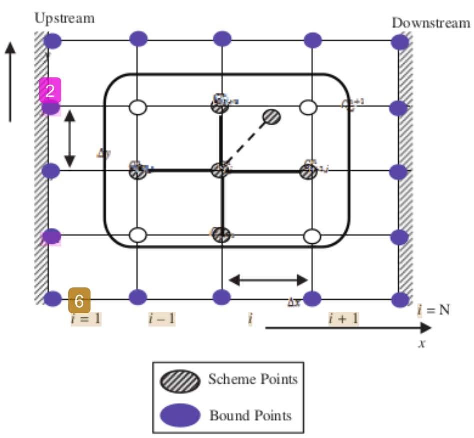

Fig. 1. Discretization pond with FTCS scheme.

Derivative approximation with respect to time with a forward finite difference approach is as follows:[10]

$$\frac{\partial C}{\partial t}(i\Delta x, j\Delta y, n\Delta t) = \frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t} - O(\Delta t) \tag{2}$$

Then, the derivatives space with a central finite difference approach are as follows:[10]

$$\frac{\partial C}{\partial x}(i\Delta x, j\Delta y, n\Delta t) = \frac{C_{i+1,j}^{n} - C_{i-1,j}^{n}}{2\Delta x} - O(\Delta x^2) \tag{3}$$

$$\frac{\partial^2 C}{\partial x^2}(i\Delta x, j\Delta y, n\Delta t) = \frac{C_{i+1,j}^{n} - 2C_{i,j}^{n} + C_{i-1,j}^{n}}{\Delta x^2} + O(\Delta x) \tag{4}$$

$$\frac{\partial^2 C}{\partial y^2}(i\Delta x, j\Delta y, n\Delta t) = \frac{C_{i,j+1}^{n} - 2C_{i,j}^{n} + C_{i,j-1}^{n}}{\Delta y^2} + O(\Delta y) \tag{5}$$

3. RESULTS AND DISCUSSION

Pollutant distribution model of advection-diffusion in this research its use is limited, because it can only be used in closed flows (pond). Waste water flows in one direction from the inlet and exit through the outlet at constant velocity.

To solve the Eq. (1), it is necessary the initial and boundary conditions. The initial conditions is:

$$C(x, y, 0) = k, \quad 0 < x < 1, \quad 0 < y < 1 \tag{6}$$

Then the boundary conditions used are Dirichlet boundary conditions, namely:

$$\begin{aligned} C(0, y, t) &= k_1, & 0 < y < 1, & \quad t > 0 \\ C(1, y, t) &= k_2, & 0 < y < 1, & \quad t > 0 \\ C(x, 0, t) &= k_3, & 0 < x < 1, & \quad t > 0 \\ C(x, 1, t) &= k_4, & 0 < x < 1, & \quad t > 0 \end{aligned} \tag{7}$$

In all cases, $k \neq k_1 \neq k_2 \neq k_3 \neq k_4$ and there is a difference, namely:

Case I, value of $k, k_1, k_2, k_3, k_4 > 30$.
Case II, value of $k \leq 30$ and $k_1, k_2, k_3, k_4 > 30$.
Case III, value of $k, k_1, k_2, k_4 \leq 30$ and $k_3 > 30$.
Case IV, value of $k, k_1, k_2, k_4 \leq 30$ and $k_3 > 30$ with some of the grid at the upper boundary has a value $k > 30$.

Equation (1) is converted into a discrete form using the finite difference method FTCS scheme, by substituting the Eqs. (2)–(5) to the the Eq. (1), so that the results are as follows:

$$\begin{aligned} \frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t} &= -u\frac{C_{i+1,j}^{n} - C_{i-1,j}^{n}}{2\Delta x} + D_{mx}\frac{C_{i+1,j}^{n} - 2C_{i,j}^{n} + C_{i-1,j}^{n}}{\Delta x^2} \\ &\quad + D_{my}\frac{C_{i,j+1}^{n} - 2C_{i,j}^{n} + C_{i,j-1}^{n}}{\Delta y^2} \end{aligned} \tag{8}$$

By algebraic manipulations, the Eq. (8) can be written as follows:

$$\begin{aligned} C_{i,j}^{n+1} &= C_{i,j}^{n}[1 - 2B_x - 2B_y] + C_{i+1,j}^{n}\left[-\frac{1}{2}A + B_x\right] \\ &\quad + C_{i-1,j}^{n}\left[\frac{1}{2}A + B_x\right] + C_{i,j+1}^{n}[B_y] + C_{i,j-1}^{n}[B_y] \end{aligned} \tag{9}$$

Using the Eq. (9) will be calculated the value of $C_{i,j}^{n+1}$ using a known point value as in Figure 1 to the desired iteration. Because

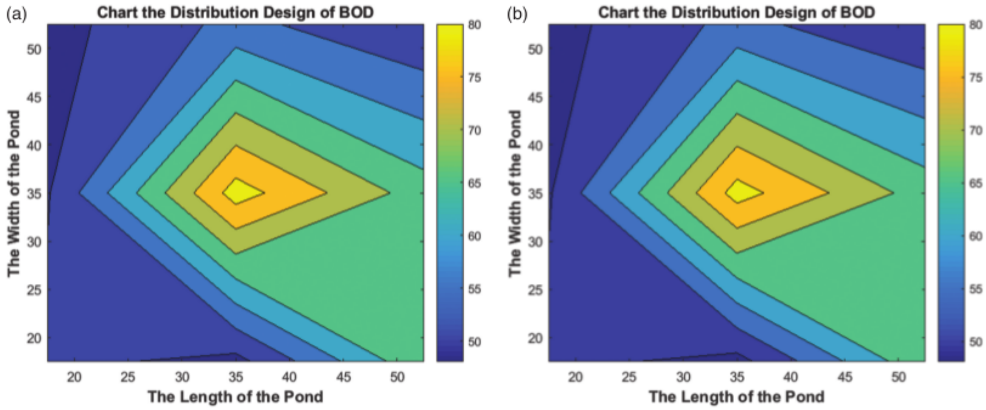

Fig. 2. Graph the distribution design of BOD of cases I (a) $t = 1$, (b) $t = 24$.

of the calculation process is very long, it will be solved with the help of MATLAB.

To solve the Eq. (9), given the values of parameters of diffusion coefficient $D_{mx} = 0.01$ m^2/s, $D_{my} = 0.01$ m^2/s, the flow velocity $u = 0.0125$ m/s and the time flow $t = 24$ hours. The given size grid $\Delta x = 19.25$, $\Delta y = 17.25$ and $\Delta t = 1$. Determination of the grid is adapted to the size of the pond, the length pond 77 meters and the width pond 70 meters.

The simulation results obtained are as follows:

Case I: Case I assumed that the BOD concentration at all of pond side and an initial value for the middle of the pond is still relatively high, namely 50–100. From the Figure 2, it can be seen that the BOD concentration decreased, but tend to be insignificant. However, a decrease in some of the grid shows that there are process of advection and diffusion in the pond.

Case II: In this case, it is assumed that the initial concentration of BOD at the observation point was appropriate with quality standards, but the concentration on all of pond side is still relatively high. The concentration in the middle of the pond is estimated appropriate with quality standards because higher diffusion process. This is due to stabilization ponds using aeration systems with completely mixed reactor at the middle of the pond.

Diffusion process in Figure 3 is more obvious than Figure 2. Decreased BOD concentration in some grids are also obvious as time passes. But in this case, the overall water has the appropriate quality standards. This is impossible to occur in waste stabilization ponds, especially at $t = 1$.

Case III: Then, Case III assumed that the BOD concentration at the pond side of the left, right, down, and an initial value for the middle of the pond has appropriate quality standards. However, because the pollutants originally came from the top side of the pond, so it is assumed that the BOD concentration at the top side of the pond is still relatively high.

The conclusion which can be obtained from observation of Figure 4 is similar to Figure 3, which is a decrease in the concentration of BOD in some grid over time. However, Figure 4 is not more obviously than Figure 3. In this case, also the overall water has the appropriate quality standards. This is impossible to occur in waste stabilization ponds, especially at $t = 1$.

Case IV: In the case of IV assumptions similar to Case III. But in this case, the BOD concentration values for some of the grid

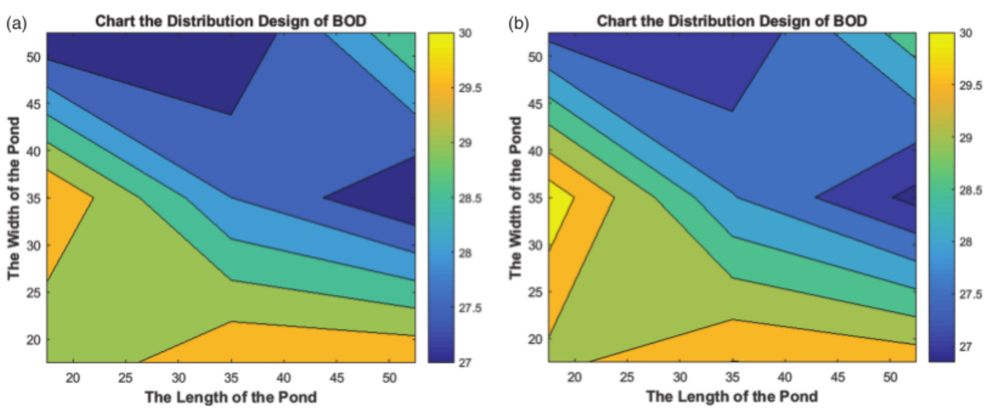

Fig. 3. Graph the distribution design of BOD of cases II (a) $t = 1$, (b) $t = 24$.

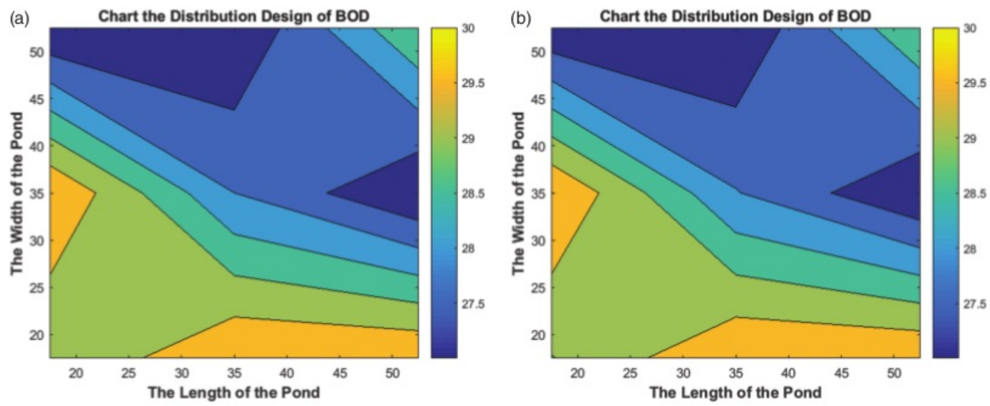

Fig. 4. Graph the distribution design of BOD of cases III (a) $t = 1$, (b) $t = 24$.

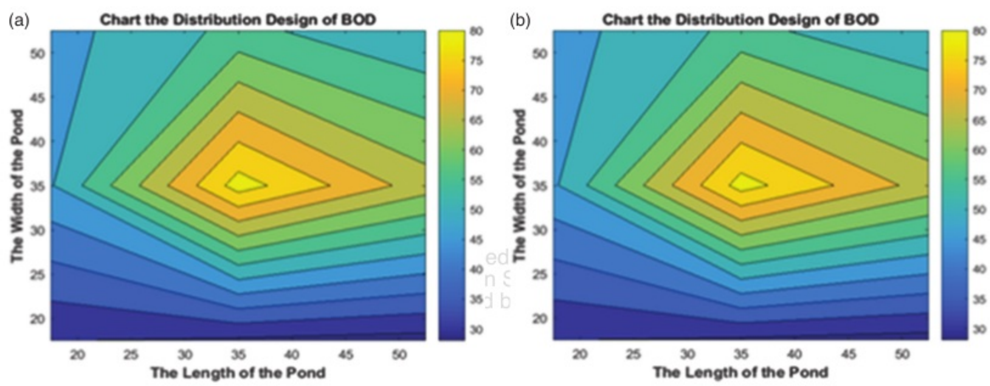

Fig. 5. Graph the distribution design of BOD of cases IV (a) $t = 1$, (b) $t = 24$.

in the middle of the pond there are still relatively high. From Figure 5, it can be seen that the BOD concentration decreased insignificantly so that tend to look the same.

4. CONCLUSION

Based on the results and discussion, it is known that the spread of BOD in waste stabilization ponds either with aeration process or not is the same. BOD in waste stabilization ponds transported (advection) and decreased exponentially with time (diffusion). Values BOD concentration on some grid decreases over time. Decrease in BOD concentration is caused by the diffusion process. It can also be concluded that the cases I and IV more logical than the cases II and III. This is due in the first case and the fourth case, initially the BOD concentration is relatively high and slowly spread up to was appropriate with the quality standards. Then in the cases of II and III, the BOD concentration from beginning to end appropriate with quality standards.

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