Numerical Solution of Distribution Model 2-D of Concentration on Chemical Oxygen Demand in Waste Stabilization Ponds

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Numerical Solution of Distribution Model 2-D of Concentration on Chemical Oxygen Demand in Waste Stabilization Ponds

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This paper presents solution of *Chemical Oxygen Demand* (COD) concentration distribution model on the surface waste stabilization ponds based on the advection-diffusion mechanism. This model is represented in second-order partial differential equations. The purpose of this paper is investigate the COD distribution design in waste stabilization ponds using field data obtained. Collection of field data was carried out in the Waste Water Treatment Plant (WWTP) Sewon, Bantul. Numerical method used for solution this model is finite difference method with Dufort Frankel scheme. The initial step in this method is process discretization by the finite difference schemes are used. The discrete equation will be substituted into the partial differential equations. Furthermore, the calculation will be completed with the help of MATLAB program. The results show that there was a mass transfer of pollutants each time followed by an increase and decrease in mass. This shows that there is a advection and diffusion process in waste stabilization ponds.

Keywords: Advection-Diffusion, Dufort Frankel, Finite Difference, Waste Stabilization Ponds.

1. INTRODUCTION

Waste stabilization ponds is biological treatment process commonly used in Indonesia. These treatment process uses an activity microorganisms to decompose the pollutant substances which present in the wastewater. The presence of microorganisms in the ponds is not accidental or man-made, but it due to the inclusion wastewater into the pool with a long residence time in accordance with the specified degree of processing.[1]

In waste stabilization ponds, wind energy and gravity provide the movement of wastewater that led to the mass transport process. The context of the movement in the waste stabilization pond system can be divided into two general mechanisms, namely advection and diffusion. Advection is a mechanism that moving a substance or material from one position to another in space, or in other words referring to mass displacement by the fluid velocity. While diffusion related to the mass movement due to the random motion of water molecules.[2] With the pollutant gradient horizontally and vertically in columns pond, the phenomenon of solute transport by advection and diffusion mechanisms can be described in 2-D models. Mathematical model of 2-D advection-diffusion is represented by using partial differential equations based on the phenomenon of physics, chemistry and biology.

The various studies have been conducted about advection diffusion mechanism with analytically and numerically solution, such as Ref. [3] developed a numerical model to predict the flow and transport of fecal matter into the water surface. Then Ref. [4] is comparing some finite difference methods, namely FTCS, Upstream, Dufort Frankel and Crank Nicolson to solve equations one-dimensional advection dispersion. Reference [5] complete the advection diffusion equation 2-D by using the finite difference method Dufort Frankel, and then Ref. [6] was combine Dufort Frankel with various other finite difference method. Reference [7] completed the advection–dispersion equation by using the three-dimensional finite difference method Forward Time Central Space (FTCS). Furthermore, Ref. [8] develop and compare several different numerical techniques to solve advection–diffusion equations in three dimensions.

Many research in waste stabilization ponds have also been carried out by the author, such as we are developed dynamic models on a facultative stabilization ponds with the phenomenon of the wastewater treatment process and then the model was used as a performance evaluation methods to Waste Water Treatment Plant (WWTP).[9] Then we did the modeling environment in the domestic waste stabilization ponds facultative with the steady state.[10] Furthermore, we are developed the distribution model 1-D organic material that does not consider the depth of the ponds. Because of the wastewater treatment using COD as indicator to

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determine the quality of wastewater, so we conducted research on the distribution model 2-D of the concentration COD on waste stabilization ponds by advection–diffusion mechanism. In this case, the COD concentration on horizontally and vertically was studied their changes through time. Changes in 2-D COD concentration through time can be represented in the form of partial differential equations. And because of the research was conducted in the stabilization ponds which are domain of simple field with a structured grid discretization process, so the numerical method used for the simulation of the model are finite difference method Dufort Frankel. Then, we use the data measurement of the concentration of the organic matter in the WWTP, Sewon, Bantul, Yogyakarta to simulation data.

2. EXPERIMENTAL DETAILS

2.1. Distribution Model of Pollutant Advection-Diffusion

Development of mathematical equations COD concentration which set by a domain can be described as advection–diffusion equation was formulated 2-D as follows:[11]

$$\frac{\partial C}{\partial t} = -\frac{\partial (uC)}{\partial x} + D_{mx}\frac{\partial^2 C}{\partial x^2} + D_{my}\frac{\partial^2 C}{\partial y^2} \qquad (1)$$

2.2. Finite Difference Method Dufort Frankel

In general, basic finite difference method is to get the value of a variable as a function of space at step-time $t+\Delta t$ by space distribution space at step-time to t known. The initial value is at zero step-time.

Data measurement of the concentration of organic matter was derived from the facultative stabilization ponds WWTP, Sewon, Bantul. Samples are measured on a facultative stabilization ponds II. Sampling points to be measured are; 9 points in the middle of the ponds with 9 points to the initial value, 16 points along the left, right, upper and bottom side of ponds. Discretization a Dufort Frankel scheme shown in Figure 1.

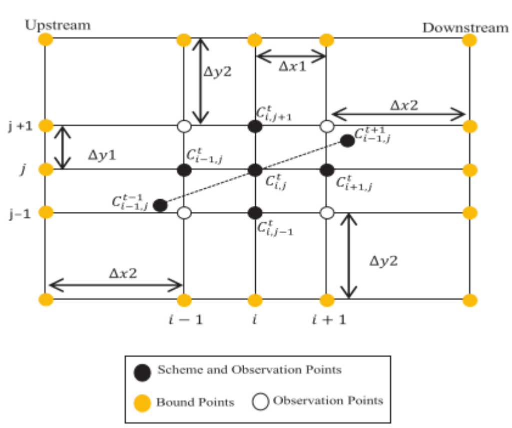

Fig. 1. Discretization ponds with Dufort Frankel scheme.

Derivative approximation with respect to time with a center finite difference approach is as follows:[12]

$$\frac{\partial C}{\partial t}(i\Delta x, j\Delta y, t\Delta t) = \frac{C^{t+1}_{i,j} - C^{t-1}_{i,j}}{\Delta t} - O(\Delta t) \qquad (2)$$

Then, the derivatives space with a center finite difference approach are as follows:[12]

$$\frac{\partial C}{\partial x}(i\Delta x, j\Delta y, t\Delta t) = \frac{C^{t}_{i+1,j} - C^{t}_{i-1,j}}{2\Delta x} - O(\Delta x^2) \qquad (3)$$

$$\frac{\partial^2 C}{\partial x^2}(i\Delta x, j\Delta y, t\Delta t) = \frac{C^{t}_{i+1,j} - C^{t+1}_{i,j} - C^{t-1}_{i,j} + C^{t}_{i-1,j}}{\Delta x^2} + O(\Delta x) \qquad (4)$$

$$\frac{\partial^2 C}{\partial y^2}(i\Delta x, j\Delta y, t\Delta t) = \frac{C^{t}_{i,j+1} - C^{t+1}_{i,j} - C^{t-1}_{i,j} + C^{t}_{i,j-1}}{\Delta y^2} + O(\Delta y) \qquad (5)$$

3. RESULTS AND DISCUSSION

Equation (1) is converted into a discrete form using the finite difference method Dufort Frankel scheme, by substituting the Eqs. (2)–(5) to the Eq. (1), so that the results are as follows:

$$\frac{C^{t+1}_{i,j} - C^{t}_{i,j}}{\Delta t} = -u\frac{C^{t}_{i+1,j} - C^{t}_{i-1,j}}{2\Delta x} + D_{mx}\frac{C^{t}_{i+1,j} - C^{t+1}_{i,j} - C^{t-1}_{i,j} + C^{t}_{i-1,j}}{\Delta x^2} + D_{my}\frac{C^{t}_{i,j+1} - C^{t+1}_{i,j} - C^{t-1}_{i,j} + C^{t}_{i,j-1}}{\Delta y^2} \qquad (6)$$

Table I. The initial value COD (mg/L).

The Grid	The initial value $t=0$	The initial value $t=1$
$C_{2,2}$	154.43	151.98
$C_{2,3}$	124.69	125.81
$C_{2,4}$	134.71	131.80
$C_{3,2}$	101.38	100.17
$C_{3,3}$	158.60	127.50
$C_{3,4}$	99.05	98.37
$C_{4,2}$	149.70	148.24
$C_{4,3}$	117.46	118.65
$C_{4,4}$	120.81	121.26

Table II. The value of upper and lower boundary.

The upper boundary	The value	The lower boundary	The value
$C_{2,1}$	195.77	$C_{2,5}$	195.77
$C_{3,1}$	143.05	$C_{3,5}$	143.05
$C_{4,1}$	174.56	$C_{4,5}$	174.56

Table III. The value of upper and lower boundary.

The right boundary	The value	The left boundary	The value
$C_{5,2}$	170.60	$C_{1,2}$	178.52
$C_{5,3}$	139.83	$C_{1,3}$	177.82
$C_{5,4}$	116.13	$C_{1,4}$	174.56

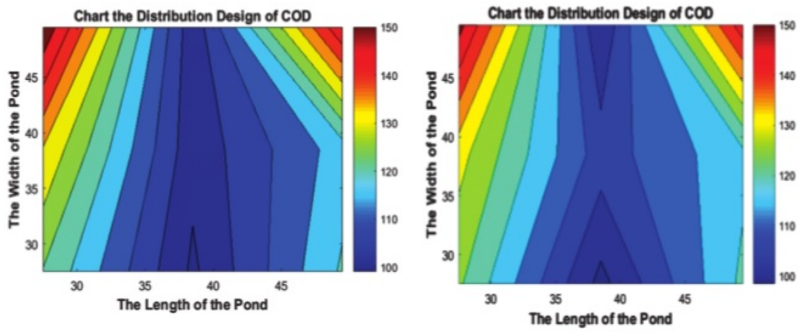

Fig. 2. Graph the distribution design of COD at $t = 2$ (left) and $t = 3$ (right).

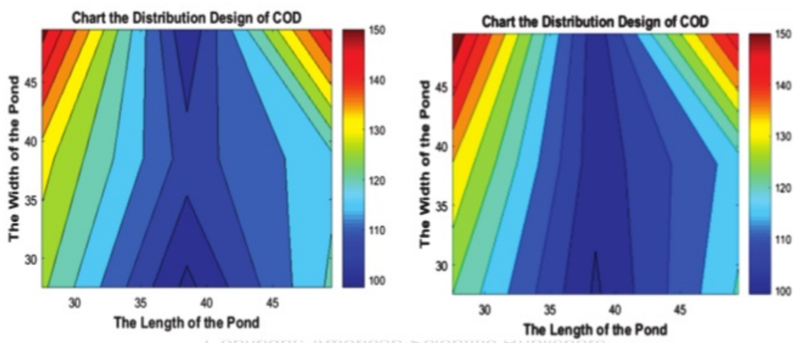

Fig. 3. Graph the distribution design of COD at $t = 23$ (left) and $t = 24$ (right).

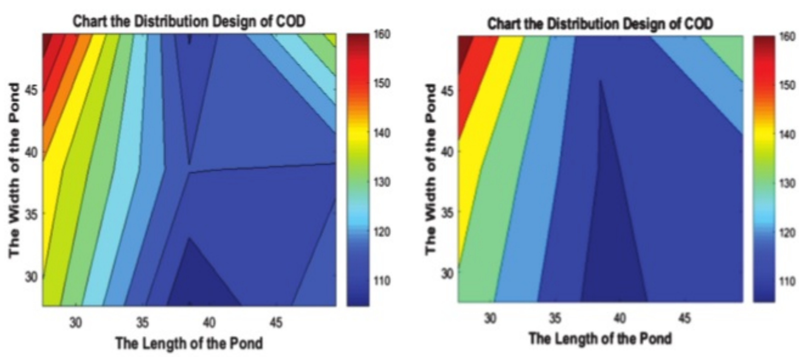

Fig. 4. Graph the distribution design of COD at $t = 999$ (left) and $t = 1000$ (right).

By algebraic manipulations, the Eq. (6) can be written as follows:

$$\begin{aligned} C_{i,j}^{t+1} &= C_{i-1,j}^{n}\frac{[A+B_x]}{[1+B_x+B_y]} + C_{i,j-1}^{t}\frac{[B_y]}{[1+B_x+B_y]} \\ &\quad + C_{i,j}^{t-1}\frac{[1-B_x-B_y]}{[1+B_x+B_y]} + C_{i,j+1}^{t}\frac{[B_y]}{[1+B_x+B_y]} \\ &\quad + C_{i+1,j}^{t}\frac{[-A+B_x]}{[1+B_x+B_y]} \end{aligned} \tag{7}$$

Using the Eq. (7) will be calculated the value of $C_{i,j}^{t+1}$ using a initial and boundary value as in Figure 1 to the desired iteration. Because of the calculation process is very long, it will be solved with the help of MATLAB. The initial value can be seen in Table I. And then, the boundary value can be seen in Tables II and III. The boundary condition which used Dirichlet boundary condition. In this case, the boundary conditions assumed to be isolated, so that the boundary conditions of constant every time.

To solve the equation, given the values of parameters of diffusion coefficient $D_{mx} = 0{,}01$ m^2/s $D_{my} = 0{,}01$ m^2/s the flow velocity $u = 0.0125$ m/s and the time flow $t = 24$ hours. The given size grid $\Delta x1 = 11$, $\Delta x2 = 27.5$, $\Delta y1 = 10$, $\Delta y2 = 25$ and $\Delta t = 1$. Determination of the grid is adapted to the size of the pond, the length pond 77 meters and the width pond 70 meters. The simulation results obtained are as follows.

From Figure 2, it can be known that every time there is a difference of movement the pollutants. This shows that the pollutants have a process of advection. From Figures 2 and 3, it can be concluded that COD has the same design every time the spread of it. However COD has a different concentration levels of each of its design, but the difference was relatively small for a matter of hours. This can be seen clearly if the period is enlarged, e.g., 1000 hours. Figures 3 and 4 represent that the concentration of COD changed, both a decrease and an increase in some of the grid. This shows that the pollutants have a process of diffusion.

4. CONCLUSION

Based on the results and discussion, it is known that pollutants in the waste stabilization ponds have advection and diffusion processes. The design of the spread of pollutants have the difference at any time or can be said to occur mass transfer of pollutants (advection). Then, there is a decrease and an increase in pollutant each advection. The increase in pollutants is thought to occur due to the aerator in the waste stabilization ponds.

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