2018 International Conference on Robotics, Biomimetics, and Intelligent Computational Systems (Robionetics)
Bandung, Indonesia, August 08-10, 2018

Single Product Inventory Control Considering Unknown Demand Using Linear Quadratic Gaussian

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***Abstract*— In this paper, we apply an existing control method i.e. linear quadratic Gaussian (LQG) for control purposes of product stock level of single product inventory system considering unknown demand. The value of this unknown demand is approached by a random variable that set to be the disturbance in the used dynamical model. The control purposes is bring the inventory point level to some trajectory reference decided by the decision maker with minimal effort (purchase cost and holding cost). From the performed computational simulation with randomly generated data, the optimal decision which is the optimal purchased product volume for whole time periods was determined and the inventory point was followed the decided reference trajectory.**

***Keywords*— inventory control, linear quadratic Gaussian, random demand**

I. INTRODUCTION

Optimal decision making on supply chain management has been developed by researchers as well as practitioners to improve the gained benefit. According to the basic definition, supply chain management is a problem of managing the relationship for maximizing the gained outcome of all parties in the related chain [1]. There are many parties/components in a supply chain network that can be supplier, carrier, manufacturer, warehouse, retailer, and at the end of chain, customer [2]. Commonly the optimal decision is made by minimizing the total occurring cost in the chain which it can be consisting of procurement cost including purchasing cost, transport cost, production cost, and holding cost if there are some raw material or product that is stored in the inventory/warehouse system, distribution cost, and many more. And surely, there are some constraints that have to be met on the optimal decision making process that can be demand satisfaction constraint, capacity constraint, computational time constraint, computation budget constraint, etc. In the manufacturer level, there are raw material warehouse that storing some raw material that will be used to produce some kind of products. In the retailer level, it will be facing a problem how to manage the stored product in their warehouse. These kinds of problem of managing the stored product in warehouse can be called inventory control problem. We focus on this kind of problem in this article. There are so many researches that were conducted to find the optimal decision for inventory control problem. Many of them are using mathematical dynamics approach to view and analyze the inventory level time by time period. The basic mathematical dynamical system for inventory system was formulated in [3]. A little bit mode advance model was formulated for multi-supplier case available in [3]–[6] and for multi-product case in [7], [8]. Furthermore, there are several advance models such as considering discount on holding cost, purchasing cost, or other cost elements [9]–[12].

The existing discussions about inventory control problem solving are commonly to regulate or track a trajectory of the inventory level and for known demand it was solved by using quadratic optimal control method such was discussed in [13]. In system and control theory, one of several classical control methods that has been widely used in many practical problem areas is Linear Quadratic Gaussian (LQG) which is working by minimizing a quadratic objective function as performance index subject to the corresponding dynamical system driven by Gaussian white noise [14]. For linear dynamical system with unknown term (or disturbance), LQG control method can be used as alternative method to solve. It is a powerful optimal control method to handle linear system considering disturbance term and it was applied to solve many problems like power system [15] and mechanical systems [16], [17]. Furthermore, it was successfully applied to solve many control problems like line-active power flow control [18], fault tolerant tracking control of UAV [19], simplified car controller design [20], dynamic information acquisition process [21] and slung load transportation problem [22].

In this paper, we have applied the LQG control method to control the inventory level of a single product inventory system where the demand is random by determining the optimal decision which is the product volume that should be purchased for each review time period. Computational simulation is performed to illustrate the evolution of the inventory level and its optimal decision.

II. DYNAMICAL SYSTEM

The formulated dynamical system works for single product case, where the value of demand is unknown. Let k denotes the time instant or time period of review on this inventory system, $y(k) \geq 0$ denotes the stock/inventory level at time period k and $d(k)$ denotes the demand for time period k. Let $u(k)$ be the amount of arriving shipment of the product that arrived at review period k and there is lead time delay between ordering

978-1-5386-6086-7/18/$31.00 ©2018 IEEE

time and arriving shipment product that denoted by n_p. By aassuming the initial inventory level is empty that is for $k < 0$, $y(k) = 0$, then the dynamics of inventory level y for any review period $k \geq 0$ can be formulated as

$$y(k) = \sum_{j=0}^{k-n_p-1} u(j) - \sum_{j=0}^{k-1} d(j). \quad (1)$$

Let say $n = n_p + 1$,

$$y(k) = x_1(k)$$

and

$$x_i(k) = u(k - n_p + i) \text{ for } i = 2,3,\ldots,n_p+1.$$

Then the dynamical of y in (1) can be rewritten as the following linear time invariant dynamical system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) - d(k) \end{aligned} \quad (2)$$

where $x(k) = [x_1(k), x_2(k), \ldots, x_n(k)]'$ as the state with constant coefficient matrices

$$A = \begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, C = [1,0,0,\cdots,0].$$

In this system, the value of demand at any review time period k is unknown but it will be approached by using some random variable that stated as the disturbance term in the LQG control method.

III. Linear Quadratic Gaussian Control Method (Review)

Consider plant model of inventory system (2) where the random demand d is approached as a noise process with $E\left[d(k)d(k)^T\right] = D(k) > 0$ as its covariance matrix. LQG is working by minimizing the following quadratic objectives

$$J = \mathrm{E}\left[x'(N)Fx(N) + \sum_{k=0}^{N-1}\left(\|x(k)\|^2_{Q(k)} + \|u(k)\|^2_{R(k)} \right) \right] \quad (3)$$

subject to (2) where $F \geq 0$, $Q(k) \geq 0$ and $R(k) \geq 0$ are denoting the weighting matrices that are symmetric and positive definite, and notation $E[\cdot]$ means the expectation value and the notation $\|v\|^2_Q$ means v^TQv. The optimal decision u is determined by finding the Kalman gain and using feedback scheme

$$u(k) = -K(k)\hat{x}(k) \quad (4)$$

where

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L(k+1)\{y(k+1) - C(A\hat{x}(k) + Bu(k))\},$$

$$K(k) = (B'S(k+1)B + R(k))^{-1} B'S(k+1)A,$$

and

$$L(k) = P(k)C'(CP(k)C' + D(k))^{-1}$$

where $P(k)$ and $S(k)$ are determined by

$$P(k+1) = A\left\{P(k) - P(k)C'(CP(k)C' + D(k))^{-1} CP(k)\right\}A'$$

$$P(0) = \mathrm{E}\left[(x(0) - \hat{x}(0))(x(0) - \hat{x}(0))'\right]$$

$$S(k) = A'\left(S(k+1) - S(k+1)B(B'S(k+1)B + R)^{-1} B'S(k+1)\right)A + Q(k)$$

with initial value $\hat{x}(0) = \mathrm{E}[x(0)]$ and $S(N) = F$.

IV. Computational Simulation

Consider a single inventory system model (2) with delay lead time 2 time periods, then the dynamical model of this inventory system is

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x(k) - d(k) \end{aligned} \quad (5)$$

where d is the random demand. The parameter values for this simulation are given by TABLE I.

TABLE I. Parameter Value

Par.	Q	R		u_{min}	u_{max}	y_{min}	y_{max}
Val.	$diag[2,2,2]$	1		0	200	0	300

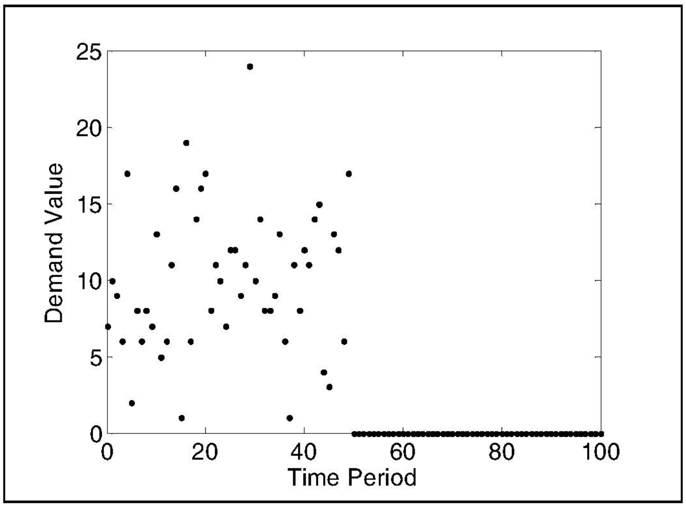

Fig. 1. Random Demand Value

Fig. 1 shows the demand value generated randomly by MATLAB (for time periods 1 to 50). For time periods 51 to 100, the demand value is made to be zero in order to analyze the generated input and output value compared to the set point value.

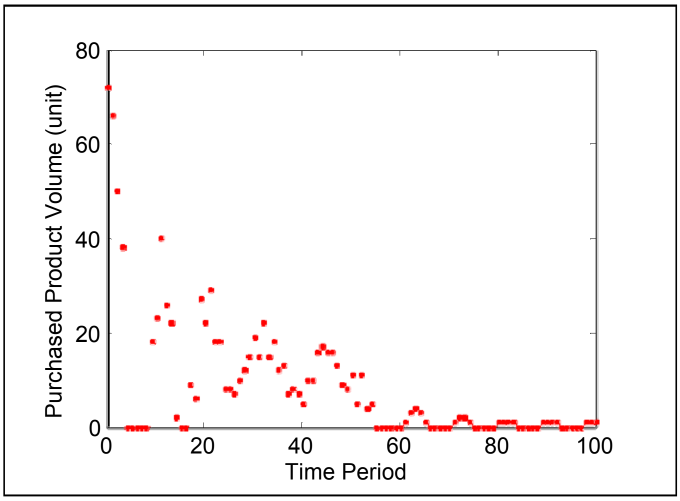

Fig. 2. Optimal input: purchased product unit $u(k)$

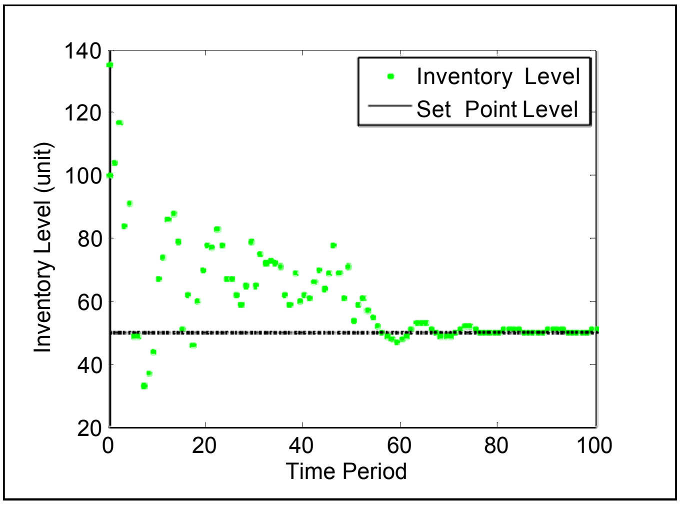

Fig. 3. Output: inventory/stored product level $y(k)$ and its reference trajectory/set point y_d

Let the inventory level at initial review time period is 100 units. Assume in this simulation the decision maker was decided that the set point level is 50 units. We simulate for 100 time periods where for time periods 1 to 50 the random demand value is assumed to be normally distributed with mean 10 and variance 5 and for time periods 51 to 100 the demand value is 0. The demand value is shown in Fig. 1 as the optimal decision u^* i.e. the purchased product volume is shown in Fig. 2. The inventory level $y(k)$ is shown in Fig. 3.

Fig. 2 shows us the optimal purchased product volume for time periods 1 to 100. From Fig. 2, it can be derived that the purchased product volume 72 units at time period 1, 66 units at time period 2, 50 units at time period 3 and so on. If this optimal purchased product volume is applied to the system, then it produces the inventory level shown in Fig. 3. Fig. 3 shows us the result of this inventory control system i.e. the stored product in the warehouse. The initial inventory level is 100 units. At time period 1, the inventory level is 134 units, at time period 2 the inventory level is 104, and so on. In the beginning of time periods 50 to 100, the inventory level was closed to the set point level i.e. 50 units where the purchased product volume was closed to 0. It was caused by the demand value for time periods 50 to 100 is 0. From these results, it can be observed that the used LQG control method was successfully worked for inventory controlling.

V. Concluding Remarks

In this article, an LQG control method was applied to determine the optimal volume of purchasing units for single product inventory system where the demand value is unknown that approached using random variable. From the simulation experiment results, the control method was successfully determined the optimal decision which is the optimal purchased product volume for all time periods where this decision gave the minimal expected total effort and the product stored in the warehouse was sufficiently closed to the decided set point as wanted by the decision maker.

In our future works, we will apply the LQG control method for multi-product inventory system. Other control methods like robust control method is also will be applied to solve this inventory control problem and the comparison between them will also be discussed.

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