Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

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To cite this article: Widowati *et al* 2018 *J. Phys.: Conf. Ser.* **1025** 012088

Stability analysis of the phytoplankton effect model on changes in nitrogen concentration on integrated multi-trophic aquaculture systems

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Abstract. Integrated Multi-Trophic Aquaculture (IMTA) is a polyculture with several biotas maintained in it to optimize waste recycling as a food source. The interaction between phytoplankton and nitrogen as waste in fish cultivation including ammonia, nitrite, and nitrate studied in the form of mathematical models. The form model is non-linear systems of differential equations with the four variables. The analytical analysis was used to study the dynamic behavior of this model. Local stability analysis is performed at the equilibrium point with the first step linearized model by using Taylor series, then determined the Jacobian matrix. If all eigenvalues have negative real parts, then the equilibrium of the system is locally asymptotic stable. Some numerical simulations were also demonstrated to verify our analytical result.

Keywords: IMTA, polyculture dynamic models, stability, equilibrium point.

1. Introduction

The IMTA cultivation is one of the efforts to conduct cultivation activities with organic reinforcement in fisheries. IMTA is available to be an ecologically well-balanced aquaculture practice from various trophic levels to optimize the recycling of agricultural waste as a food source [1]. IMTA is an aquaculture practice using more than one species of biotas which have ecologically mutual relationship as a part of the food chain in the area at the same time [2]. The benthic nutrient fluxes and total organic carbon (TOC) of sediment in IMTA system was significantly lower than in monoculture bays which is mainly controlled by seawater temperature, dissolved oxygen (DO) and nutrient concentrations, which were strongly related to aquaculture activities [3]. Phytoplankton in IMTA provides an important role because it has chlorophyll to perform photosynthesis. Through photosynthesis process, phytoplankton can reduce the levels of ammonia, nitrite, and nitrate in IMTA. The interaction of phytoplankton in cultivation is related to its role in reducing the concentration of ammonia, nitrite, and nitrate, which can be written in a dynamic system.

In general, dynamic system is defined as a real problem that is modeled in the form of differential equations. One example of dynamic systems is a system of ammonia, nitrite, and nitrate concentration changes in the fish culture [4]. Ammonia is a waste in aquaculture water which is the result of bacterial decomposition of the unfeed pellets and fish faeces and urine. Furthermore, through the process of nitrification, ammonia turn into nitrites and nitrates. Nitrite as a result of ammonia oxidation is also an inorganic nitrogen compound, which can endanger the life of fish when presents in high quantities [5].

Previous research using macrobenthos samples are taken from sediment using Eckman catch, then analyzed by using diversity approach and quality index. This results implied that the use of area for both polyculture and industrial activities can may lead to environmental disturbance, thus enviromental coastal management need to be applied in regular basis, both temporally and spatially [6]. The sediment samples were taken from the two locations with two sampling time and three replicates using Van Veen Grab for biotic and abiotic assessment [7]. A dynamic model of the effect of fish management on the accumulation of nitrogen in aquaculture have been developed [8]. However, the model proposed is still a simulation of physical and biogeochemical processes during the formation, and no stability analysis has been done. So, we need a mathematical model and stability analysis of the system.

2. Brief Review of the Mathematical Modeling

Several previous studies have developed mathematical models related to the concentration of nitrogen with monoculture system. So, it is a necessary mathematical model that is able to describe the concentration of nutrients with the number of marine biotas. The performance of deposit feeders in integrated multi-trophic aquaculture (IMTA) was analyzed through the application of mathematical models [9]. The nitrogen dynamic model for IMTA waters has been discussed [10] which explains the influence of growth of phytoplankton with changes in ammonia, nitrite, and nitrate concentrations. Model of nitrogen transformation in IMTA follows the Lorenzen model [11]. This model illustrates the concentration of nitrogen, and phytoplankton in an open water aquaculture system consisting of three variables, namely ammonia, NOx, and phytoplankton. In previous research, Widowati et al [12] had broken down the NOx variable into two variables, which is nitrite and nitrate.

The phytoplankton relationship model and ammonia, nitrate, and nitrate changes were initiated by the growth of phytoplankton caused by the amount of ammonia used by phytoplankton equal to ρ and nitrate of $(1-\rho)$. The assimilation of ammonia and nitrate by phytoplankton is assumed to be proportional to its relative concentration in the air. The assimilation of ammonia by phytoplankton is $\rho = \frac{N_1(\mathrm{t})}{N_1(\mathrm{t})+N_3(\mathrm{t})}$ and assimilation of nitrate is $(1-\rho) = \frac{N_3(\mathrm{t})}{N_1(\mathrm{t})+N_3(\mathrm{t})}$.

The interaction between phytoplankton and change of ammonia, nitrite, nitrate, and for simplification of the writing of dependence on the function t is omitted, the dynamic model is obtained as follows:

$$\begin{aligned}
\frac{d\mathrm{P}}{dt} &= \alpha\beta P - (a+\mu)P \\
\frac{dN_1}{dt} &= \Lambda - \delta N_1 - \alpha\beta P\left(\frac{N_1}{N_1+N_3}\right) \\
\frac{dN_2}{dt} &= k_1 N_1 - k_2 N_2 \\
\frac{dN_3}{dt} &= k_2 N_2 - a N_3 - \alpha\beta F\left(\frac{N_3}{N_1+N_3}\right)
\end{aligned} \tag{1}$$

Here P is phytoplankton concentration, N_1 is ammonia concentration, N_2 is nitrite concentration, N_3 is nitrate concentration.

Nitrogen in the form of ammonia, nitrite, and nitrate is a key element in aquatic environments and cultivation. Feeding of nitrogen in the form of food is needed to increase the aquatic production of the cultivated animals and is considered an important fish management variable. Nitrogen input in the cultivation also increases the potential for pollution to the surrounding environment [4].

3. Stability Analysis of the Model

Next, we determine the equilibrium point of the system (1). The point of equilibrium is obtained if it meets $\frac{dP}{dt}=0, \frac{dN_1}{dt}=0, \frac{dN_2}{dt}=0, \frac{dN_3}{dt}=0$, let $(P^*, {N_1}^*, {N_2}^*, {N_3}^*)$ stating the equilibrium point of system (1) so that the system can be written,

$$\frac{dP}{dt} = \alpha\beta P^* - (a+\mu)P^* = 0$$
$$\frac{dN_1}{dt} = \Lambda - \delta {N_1}^* - \alpha\beta P^*\left(\frac{{N_1}^*}{{N_1}^*+{N_3}^*}\right) = 0$$
$$\frac{dN_2}{dt} = k_1{N_1}^* - k_2{N_2}^* = 0$$
$$\frac{dN_3}{dt} = k_2{N_2}^* - aN_3 - \alpha\beta P^*\left(\frac{{N_3}^*}{{N_1}^*+{N_3}^*}\right) = 0 \tag{2}$$

Then we get $P^* = 0, N_1 = \frac{\Lambda}{\delta}, N_2 = \frac{k_1\Lambda}{k_2\delta}, N_3 = \frac{k_1\Lambda}{a\delta}$.

From the calculation of equilibrium point as follows $(P^*, {N_1}^*, {N_2}^*, {N_3}^*) = \left(0, \frac{\Lambda}{\delta}, \frac{k_1\Lambda}{k_2\delta}, \frac{k_1\Lambda}{a\delta}\right)$. The behaviour of non-linear system (1) around the point of equilibrium $(P^*, {N_1}^*, {N_2}^*, {N_3}^*)$ can be determined by linearization non-linear systems. Linearization method use the first order Taylor expansion around the equilibrium point. We obtain the following equations:

$$\frac{d\bar{P}}{dt} = \frac{\partial F_1}{\partial P}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{P} + \frac{\partial F_1}{\partial N_1}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{N}_1 + \frac{\partial F_1}{\partial N_2}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{N}_2 + \frac{\partial F_1}{\partial N_3}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{N}_3$$
$$\frac{d\bar{N}_1}{dt} = \frac{\partial F_2}{\partial P}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{P} + \frac{\partial F_2}{\partial N_1}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{N}_1 + \frac{\partial F_2}{\partial N_2}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{N}_2 + \frac{\partial F_2}{\partial N_3}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{N}_3$$
$$\frac{d\bar{N}_2}{dt} = \frac{\partial F_3}{\partial P}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{P} + \frac{\partial F_3}{\partial N_1}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{N}_1 + \frac{\partial F_1}{\partial N_2}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{N}_2 + \frac{\partial F_3}{\partial N_3}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{N}_3$$
$$\frac{d\bar{N}_3}{dt} = \frac{\partial F_4}{\partial P}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{P} + \frac{\partial F_4}{\partial N_1}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{N}_1 + \frac{\partial F_4}{\partial N_2}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{N}_2 + \frac{\partial F_4}{\partial N_3}(P^*, {N_1}^*, {N_2}^*, {N_3}^*)\bar{N}_3 \tag{3}$$

Linearization of the model (1) around the point of equilibrium $(P^*, {N_1}^*, {N_2}^*, {N_3}^*)$ by using the first order Taylor expansion is as follows:

$$\frac{d\bar{P}}{dt} = -(-\alpha\beta + a + \mu)\bar{P}$$
$$\frac{d\bar{N}_1}{dt} = \left(-\alpha\beta\frac{{N_1}^*}{{N_1}^*+{N_3}^*}\right)\bar{P} + \left(-\delta - \frac{\alpha\beta P^*}{{N_1}^*+{N_3}^*} + \left(\frac{\alpha\beta P^*{N_1}^*}{({N_1}^*+{N_3}^*)^2}\right)\right)\bar{N}_1 + \left(\frac{\alpha\beta P^*{N_1}^*}{({N_1}^*+{N_3}^*)^2}\right)\bar{N}_3$$
$$\frac{d\bar{N}_2}{dt} = k_1\bar{N}_1 - k_2\bar{N}_2$$
$$\frac{d\bar{N}_3}{dt} = \left(-\alpha\beta\frac{{N_3}^*}{{N_1}^*+{N_3}^*}\right)\bar{P} + \left(\frac{\alpha\beta P^*{N_3}^*}{({N_1}^*+{N_3}^*)^2}\right)\bar{N}_1 + k_2\bar{N}_2 + \left(-a - \frac{\alpha\beta P^*}{{N_1}^*+{N_3}^*} + \left(\frac{\alpha\beta P^*{N_3}^*}{({N_1}^*+{N_3}^*)^2}\right)\right)\bar{N}_3 \tag{4}$$

Further, we find Jacobian matrix of the system (4) as follows.

$$J=\begin{bmatrix} -(-\alpha\beta+a+\mu) & 0 & 0 & 0 \\ -\alpha\beta\dfrac{N_1^*}{N_1^*+N_3^*} & -\delta-\dfrac{\alpha\beta P^*}{N_1^*+N_3^*}+\left(\dfrac{\alpha\beta P^* N_1^*}{(N_1^*+N_3^*)^2}\right) & 0 & \dfrac{\alpha\beta P^* N_1^*}{(N_1^*+N_3^*)^2} \\ 0 & k_1 & -k_2 & 0 \\ -\alpha\beta\dfrac{N_3^*}{N_1^*+N_3^*} & \dfrac{\alpha\beta P^* N_3^*}{(N_1^*+N_3^*)^2} & k_2 & -a-\dfrac{\alpha\beta P^*}{N_1^*+N_3^*}+\left(\dfrac{\alpha\beta P^* N_3^*}{(N_1^*+N_3^*)^2}\right) \end{bmatrix}$$

Furthermore, we analyze the stability of the system by substituting the value of equilibrium point $\left(0,\frac{\Lambda}{\delta},\frac{k_1\Lambda}{k_2\delta},\frac{k_1\Lambda}{a\delta}\right)$ to the Jacobian matrix J to obtain

$$J=\begin{bmatrix} -(-\alpha\beta+a+\mu) & 0 & 0 & 0 \\ \dfrac{-\alpha\beta a}{a+k_1} & -\delta & 0 & 0 \\ 0 & k_1 & -k_2 & 0 \\ \dfrac{-\alpha\beta k_1}{a+k_1} & 0 & k_2 & -a \end{bmatrix}$$

By using $\det|\lambda I-J|=0$ and obtained characteristic equation

$$\big(\lambda+(-\alpha\beta+a+\mu)\big)(\lambda+\delta)(\lambda+k_2)(\lambda+a)=0$$

From this characteristic equation is obtained eigenvalues as follows,

$$\lambda_1=-(-\alpha\beta+a+\mu),\lambda_2=-\delta,\lambda_3=-k_2,\lambda_4=-a$$

Thus, the stability of the equation depends on the value of root characteristic. Value $\lambda_1=-(-\alpha\beta+a+\mu)<0$ for $\alpha,\beta,a,\mu>0$ and $\lambda_2=-\delta<0$ for $\delta>0$ and $\lambda_3=-k_2<0$ for $k_2>0$ and $\lambda_4=-a<0$ for $a>0$. According to the theory of stability for the eigenvalues obtained is negative or $R_e(\lambda)<0$ then the equilibrium point $\left(0,\frac{\Lambda}{\delta},\frac{k_1\Lambda}{k_2\delta},\frac{k_1\Lambda}{a\delta}\right)$ on this system is asymptotically stable.

4. Numerical Simulation

In this section a numerical simulation is given as a verification of the proposed method of research. Without prejudice to the generality for numerical simulations, we used parameter values obtained by [1,12,13], i.e $\Lambda=3.98, v=0.052, a=0.068, k_1=0{,}00187, k_2=1{,}623, \alpha=0.54, \beta=1.55, \mu=0.85$. From the data obtained by the mathematical model of non-linear differential equations system as follows,

$$\frac{dP}{dt}=-0.0810P$$
$$\frac{dN_1}{dt}=3.98-1.743N_1-(0.8370)P\left(\frac{N_1}{N_1+N_3}\right)$$
$$\frac{dN_2}{dt}=0.00187N_1-1.623N_2$$
$$\frac{dN_3}{dt}=1.623N_2-0.068N_3-(0.8370)P\left(\frac{N_3}{N_1+N_3}\right) \tag{5}$$

The equilibrium point is obtained, if it satisfy $\frac{dP}{dt}=0,\frac{dN_1}{dt}=0,\frac{dN_2}{dt}=0,\frac{dN_3}{dt}=0$, let $(P^*,{N_1}^*,{N_2}^*,{N_3}^*)$ stating the equilibrium point of system (5) so that the system as follows,

$$\frac{dP}{dt}=-0.0810P^*$$
$$\frac{dN_1}{dt}=3.98-1.743N_1-(0.8370)P^*\left(\frac{{N_1}^*}{{N_1}^*+{N_3}^*}\right)$$

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$$\frac{dN_2}{dt} = 0.00187{N_1}^* - 1.623{N_2}^*$$
$$\frac{dN_3}{dt} = 1.623{N_2}^* - 0.068{N_3}^* - (0.8370)\mathrm{P}^*\left(\frac{{N_3}^*}{{N_1}^*+{N_3}^*}\right) \tag{6}$$

From the calculation of the equilibrium point based on the data as follows $(P^*, {N_1}^*, {N_2}^*, {N_3}^*) = (0, 2.28342, 0.00263, 0.06279)$. Then, we analyze of the local stability around the equilibrium point. In this case, we find eigenvalues $\lambda_1 = -0.081$ and $\lambda_2 = -0.12187$ and $\lambda_3 = -1.623$ and $\lambda_4 = -0.068$. According to the theory of stability for the eigenvalues obtained are negative or $R_e(\lambda) < 0$ so that the equilibrium point $(P^*, {N_1}^*, {N_2}^*, {N_3}^*) = (0, 2.28342, 0.00263, 0.06279)$ on this system is asymptotically stable. This shows growth and changes in ammonia, nitrite, and nitrate levels corresponding to phytoplankton in IMTA cultivation that form a stable system.

We obtain the mathematical model (5). Based on data variables $P(0) = 0.00952$ $N_1(0) = 0.071, N_2(0) = 0.0042, N_3(0) = 0.7130$ from Riau Archipelago [14,15]. The simulations are used to see field plot of the model.

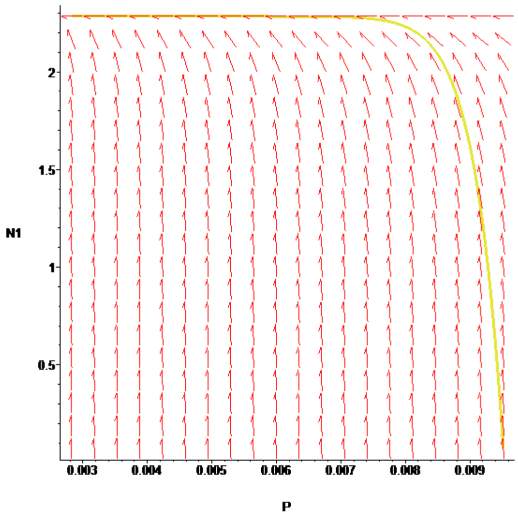

Figure 1.a. Trajectories for a system in $P - N_1$ plane

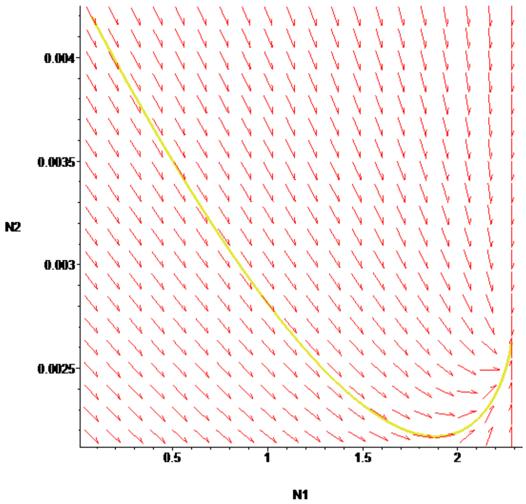

Figure 1.b. Trajectories for a system in $N_1 - N_2$ plane

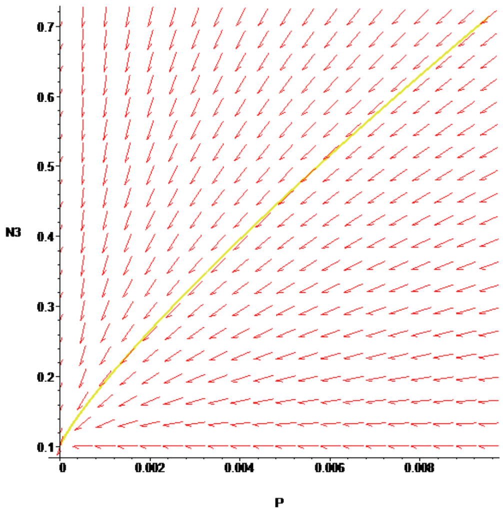

Figure 1.c. Trajectories for a system in $P - N_3$ plane

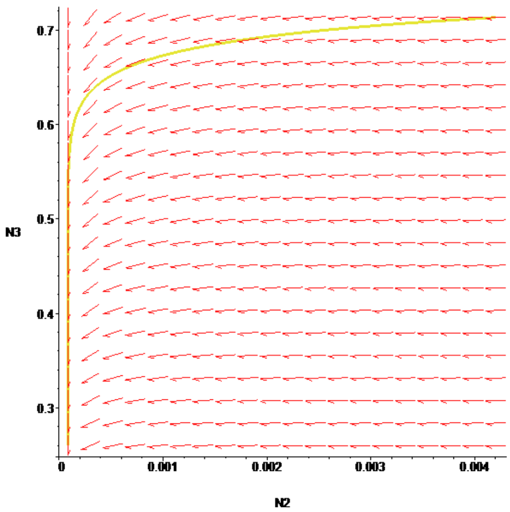

Figure 1.d. Trajectories for a system in $N_2 - N_3$ plane

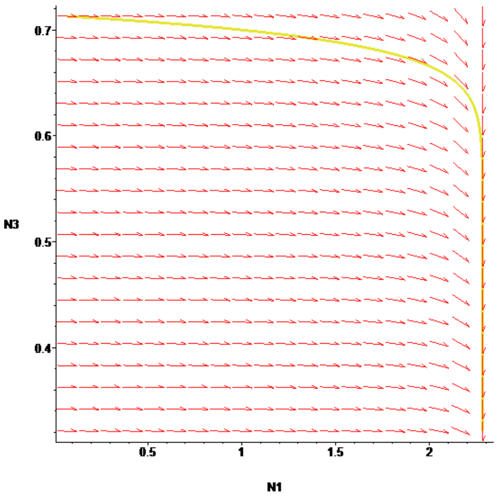

Figure 1.e. Trajectories for a system in $N_1 - N_3$ plane

From the figure 1.a – 1.e, it can be seen that the behavior of the system solution trajectory reaches to the point of equilibrium. This indicates that the equilibrium point is asymptotically stable.

5. Concluding Remark

In this paper, the discussed mathematical model is the behavior of nitrogen concentrations in IMTA associated with phytoplankton. The form of the model is a non-linear dynamic system. Based on the results of the discussion on the analysis of the stability is found that if the real part of the eigenvalues are negative so that the equilibrium point $(P^*, N_1{}^*, N_2{}^*, N_3{}^*) = \left(0, \frac{\Lambda}{\delta}, \frac{k_1\Lambda}{k_2\delta}, \frac{k_1\Lambda}{a\delta}\right)$ on the system is asymptotically stable. Further we analize, local stability of the dynamical system based the theory of stability, we find that eigenvalues are negative or $R_e(\lambda) < 0$ then the equilibrium point in the system is asymptotically stable. It means that changes in ammonia, nitrite, and nitrate concentrations are influenced by phytoplankton.

Acknowledgment

The authors thank the Directorate-General for Research, Technology and Higher Education that has funded this project through PUSNAS program, the period of 2017.

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