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Expected value based fuzzy programming approach to solve integrated supplier selection and inventory control problem with fuzzy demand

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Abstract. In this paper, a mathematical model in quadratic programming with fuzzy parameter is proposed to determine the optimal strategy for integrated inventory control and supplier selection problem with fuzzy demand. To solve the corresponding optimization problem, we use the expected value based fuzzy programming. Numerical examples are performed to evaluate the model. From the results, the optimal amount of each product that have to be purchased from each supplier for each time period and the optimal amount of each product that have to be stored in the inventory for each time period were determined with minimum total cost and the inventory level was sufficiently closed to the reference level.

1. Introduction

In logistic and supply chain management (LSCM), commonly a manufacturer faces supplier selection problem which can be presented as an optimal supplier finding to supply some raw material or product to the manufacturer with minimal total cost [1]. The most developed method to find the optimal supplier is mathematical model method in a mathematical optimization form such as mixed-integer linear programming that was developed in [2, 3] . To meet the demand for the future period, the manufacturer can store some product in the storage where the holding cost is charged. Commonly, the stored product is to be minimized in order to minimize the holding cost, but for some cases like retail manufacture, the decision maker is deciding to store the product as many he wants which can be called set point inventory level. To minimize the total cost so that the inventory level as close as possible to a set point level, a reference tracking control method is needed.

The most published articles was developed a method to solve the supplier selection with known demand value i.e. the demand value of all product at each time period is certainly measured. Unfortunately, there are many case that the demand value is unknown which can be called uncertain demand. To solve this problem, the uncertainty theory is needed. Since it was introduced in [4], possibility theory was being powerful tool to solve many problems in uncertain environment mainly in fuzzy optimization. Inspired by expected value of a random variable in probabilistic theory, the expected value of a fuzzy variable was introduced in [5] to solve a fuzzy programming. Many researchers were successfully utilized fuzzy expected value approach to solve many problems like data envelopment analysis [6 - 8], industrial grinding process [9, 10], portfolio optimization [11-13] and many more.

In this paper, we propose a mathematical model in quadratic programming with fuzzy parameter to determine the optimal strategy for integrated inventory control and supplier selection problem with fuzzy demand where the corresponding optimization will be solved by using expected value based fuzzy programming approach. Numerical experiment will be performed to evaluate the proposed model and to analyze the optimal strategy for this problem.

2. Problem Definition and Notations

Suppose that a manufacturer will solve supplier selection integrated with inventory control problem. The condition of the problem is assumed to be multi-product, multi-supplier and multi-period problem. To formulate the mathematical model, symbol of variables and parameters are introduced as follows:

- T : Set of time period;
- S : Set of supplier;
- P : Set of product;
- X_{tsp} : Volume of product $p \in P$ from supplier $s \in S$ at time period $t \in T$;
- UP_{tsp} : Unit price of product $p \in P$ from supplier $s \in S$ at time period $t \in T$;
- TC_{ts} : Transportation cost of all product from supplier $s \in S$ at time period $t \in T$;
- Y_{ts} : Supplier assignment $s \in S$ at time period $t \in T$ (1 if $X_{tsp} > 0$ exist, 0 if none);
- SC_{tsp} : Supplying capacity of supplier $s \in S$ for product $p \in P$ at time period $t \in T$;
- UPC_{tsp} : Unit penalty cost for defect product unit $p \in P$ from supplier $s \in S$ at time period $t \in T$;
- UDC_{tsp} : Unit delay cost for delayed product unit $p \in P$ from supplier $s \in S$ at time period $t \in T$;
- DLT_{tsp} : Delay lead time of product $p \in P$ from supplier $s \in S$ at time period $t \in T$;
- Q_{tsp} : Quality level of product $p \in P$ at period $t \in T$ from supplier $s \in S$;
- I_{tp} : Inventory level of product $p \in P$ at time period $t \in T$;
- M_p : Storage capacity of product $p \in P$;
- B_t : Cost budget at time period $t \in T$;
- HC_{tp} : Holding cost of product $p \in P$ at time period $t \in T$.

3. Expected Value Based Fuzzy Programming Model

The general form of an expected value based fuzzy programming with crisp objective function and fuzzy constraint can be expressed as follows

$$\begin{cases} \min f(x) \\ \text{s.t. } g_i(x,\xi) \geq 0, i = 1,2,\ldots,p. \end{cases} \tag{1}$$

where x is the decision vector, ξ is a fuzzy vector, $f(x)$ is the objective/return function and $g_i(x,\xi) \geq 0$ are constraint functions. Since the constraint do not produce a crisp feasible set, model (1) is not well-defined. Then we use expected value to approach the crisp value of the constraint. There are several ways to define the expected value of a fuzzy variable. In this paper, we use the expected value that was defined by Liu and Liu [5]. For any t and p, the expected value of fuzzy demand $\tilde{D}_{tp}$ is defined by

$$E\left[\tilde{D}_{tp}\right] = \int_0^{\infty} Cr\left\{\tilde{D}_{tp} \geq r\right\} dr - \int_{-\infty}^{0} Cr\left\{\tilde{D}_{tp} \leq r\right\} dr \tag{2}$$

provided that at least one of the two integrals is finite where $Cr[\cdot]$ denotes the credibility value. By using (2), it can be proven that the expected value of a triangle fuzzy variable (a,b,c) is

$E\left[(a,b,c)\right]=\frac{a+2b+c}{4}$ and the expected value of a trapezoidal fuzzy variable (a,b,c,d) is $E\left[(a,b,c,d)\right]=\frac{a+b+c+d}{4}$. For a discrete fuzzy variable ξ whose membership function is given by

$$\mu_\xi(x)=\begin{cases}\mu_1, \text{ if } x=x_1\\ \mu_2, \text{ if } x=x_2\\ \vdots\\ \mu_m, \text{ if } x=x_m\end{cases} \quad (3)$$

where $x_1, x_2, \dots, x_m$ are distinct numbers and $x_1<x_2<\cdots<x_m$, then the expected value of ξ is

$$E[\xi]=\sum_{i=1}^{m} w_i x_i \quad (4)$$

where

$$w_i=\tfrac{1}{2}\left(\max_{1\le j\le i}\mu_j-\max_{1\le j<i}\mu_j+\max_{i\le j\le m}\mu_j-\max_{1<j\le m}\mu_j\right) \quad (5)$$

for $i=1,2,\dots,m.$

The mathematical model for integrated supplier selection and inventory control of a multiproduct inventory system with fuzzy demand is formulated as follows. The objective function is the total procurement and inventory cost i.e.

$$\begin{aligned}Z=&\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{p=1}^{P}X_{tsp}\cdot UP_{tsp}+\sum_{t=1}^{T}\sum_{s=1}^{S}TC_{ts}\cdot Y_{ts}+\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{p=1}^{P}(1-Q_{tsp})\cdot UPC_{tsp}\cdot X_{tsp}\cdot Y_{ts}\\&+\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{p=1}^{P}UDC_{tsp}\cdot DLT_{tsp}\cdot X_{tsp}\cdot Y_{ts}+\sum_{t=1}^{T}\sum_{p=1}^{P}HC_{tp}\cdot I_{tp}+\sum_{t=1}^{T}\sum_{p=1}^{P}\left(I_{tp}-r_{tp}\right)^2\end{aligned} \quad (6)$$

where the terms in Z representing the product buying cost for all product from all supplier for all time period, the transportation cost, suppliers service level cost, product defect cost, holding cost, and inventory level tracking objective respectively. The constraints of the model are formulated as follows starting from the constraints who's containing fuzzy variable. The first constraint is the inventory management which is

$$\sum_{s=1}^{s}X_{tsp}-I_{tp}\ge\tilde{D}_{tp}, \text{ for } t=1,\forall p\in P, \text{ and } V_{t-1,p}+\sum_{s=1}^{s}X_{tsp}-V_{tp}\ge\tilde{D}_{tp}, \text{ for } t>1, t\in T,\forall p\in P. \quad (7)$$

The second constraint is presenting to determine the supplier's assignment to supply a product i.e.

$$\left(\sum_{p=1}^{P}\tilde{D}_{tp}\right)Y_{ts}\ge\sum_{p=1}^{P}X_{tsp},\forall t\in T,\forall s\in S. \quad (8)$$

The remaining constraints are formulated as follows

$$X_{tsp}\le SC_{tsp},\forall t\in T,\forall s\in S,\forall p\in P, \quad (9)$$

$$Y_{ts} = \begin{cases} 1, & \text{if } \sum_{p=1}^{P} X_{tsp} > 0 \\ 0, & \text{others} \end{cases}, \forall t \in T, \forall s \in S \tag{10}$$

$$I_{tp} \le M_{tp}, \forall t \in T, \forall p \in P, \tag{11}$$

$$\begin{aligned} &\sum_{s=1}^{S}\sum_{p=1}^{P} X_{tsp} \cdot UP_{tsp} + \sum_{s=1}^{S} TC_{ts} \cdot Y_{ts} + \sum_{s=1}^{S}\sum_{p=1}^{P} (1 - Q_{tsp}) \cdot UPC_{tsp} \cdot X_{tsp} \cdot Y_{ts} \\ &+ \sum_{s=1}^{S}\sum_{p=1}^{P} UDC_{tsp} \cdot DLT_{tsp} \cdot X_{tsp} \cdot Y_{ts} + \sum_{p=1}^{P} HC_{tp} \cdot I_{tp} + \sum_{t=1}^{T}\sum_{p=1}^{P} \left(I_{tp} - r_{tp}\right)^2 \le B_t, \forall t \in T, \end{aligned} \tag{12}$$

$$X_{tsp} \ge 0, \forall t \in T, \forall s \in S, \forall p \in P, \tag{13}$$

$$X_{tsp}, I_{tp}, \forall t \in T, \forall s \in S, \forall p \in P \text{ integer}, \tag{14}$$

where constraints (9)-(14) are presenting the supplier capacity satisfying, product quality level, supplier's assignment, non-negativity, storage capacity satisfying, budget satisfying, and integer constraint respectively. By applying expected value (2) to the fuzzy demand parameter $\tilde{D}_{tp}$, then we formulate the model as minimizing total cost (6) subject to (7)-(14) i.e.

$$\min Z \tag{15}$$

subject to:

$$\sum_{s=1}^{s} X_{tsp} - I_{tp} \ge E\left[\tilde{D}_{tp}\right], \text{ for } t = 1, \forall p \in P, \text{ and } V_{t-1,p} + \sum_{s=1}^{s} X_{tsp} - V_{tp} \ge E\left[\tilde{D}_{tp}\right], \text{ for } t > 1, t \in T, \forall p \in P;$$

$$\left(\sum_{p=1}^{P} E\left[\tilde{D}_{tp}\right]\right) Y_{ts} \ge \sum_{p=1}^{P} X_{tsp}, \forall t \in T, \forall s \in S;$$

and (9)-(14). The expected value $E\left[\tilde{D}_{tp}\right]$ can be calculated according to the definition of a membership function. The authors deal with the definition (2) to define the expected value of a fuzzy variable.

4. Numerical Examples

In this section, we evaluate the proposed model with randomly generated data which available in appendix. Suppose that we have four suppliers S1, S2, S3, S4 to supply three kind of products P1, P2, P3. The decision maker is deciding that the inventory level of each product must follow a set point as close as possible with minimum total cost.

Let the initial inventory level is 0 and the decision maker wants to determine the optimal strategy by using the model (15) for 5 time periods while the parameter values of the problem is available in appendix. We simulate the model with two different membership functions of the fuzzy demand parameter which are discrete membership function (discussed in sub-subsection 4.1) and triangular membership function (discussed in sub-subsection 4.2).

4.1. Discrete fuzzy demand

For the first example, suppose that the demand for any time period and any product is assumed to be a fuzzy variable with discrete membership function in the form of (3) that is defined by the following membership function

$$\mu_{\tilde{D}_{tp}} = \begin{cases} 0.45 \text{ if } \tilde{D}_{tp} = 300;\ 0.50 \text{ if } \tilde{D}_{tp} = 310;\ 0.65 \text{ if } \tilde{D}_{tp} = 320;\ 0.80 \text{ if } \tilde{D}_{tp} = 330;\ 0.95 \text{ if } \tilde{D}_{tp} = 340; \\ 1.00 \text{ if } \tilde{D}_{tp} = 350;\ 0.85 \text{ if } \tilde{D}_{tp} = 360;\ 0.75 \text{ if } \tilde{D}_{tp} = 370;\ 0.60 \text{ if } \tilde{D}_{tp} = 380;\ 0.55 \text{ if } \tilde{D}_{tp} = 390 \end{cases}$$

for all *t* and *p*. By using (4), the expected value of $\tilde{D}_{tp}$ is $E\left[\tilde{D}_{tp}\right] = \sum_{i=1}^{10} w_i \left(\tilde{D}_{tp}\right)_i$ where $w_1 = 0.225$, $w_2 = 0.025$, $w_3 = 0.075$, $w_4 = 0.0.075, w_5 = 0.1$, $w_6 = 0.05, w_7 = 0.075$, $w_8 = 0.025, w_9 = 0.275$, and $w_{10} = 0.275$. We solve optimization (15) by using LINGO® 16.0 in Windows Operating System with 2 GB of memory and Dual Core 1.5 GHz of processor. The solution is given in Fig. 1 and Fig. 2.

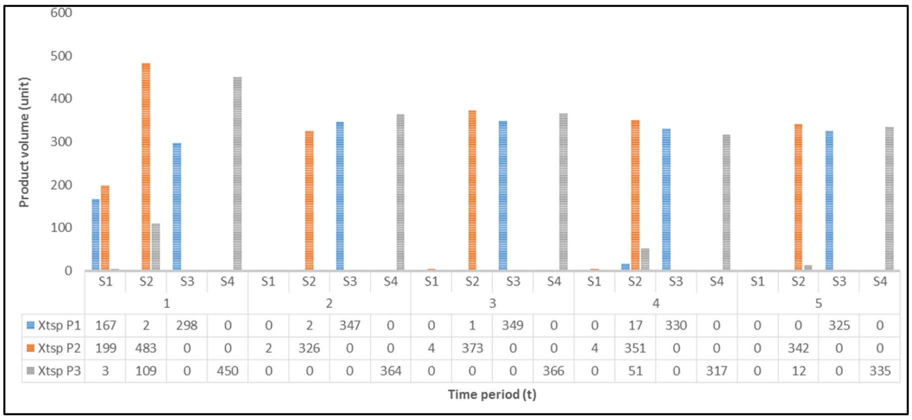

Figure 1. Optimal product volume that should be purchased from each supplier in the first example

The optimal strategy for this problem is the optimal values of $X_{tsp}, I_{tp}, \forall t \in T, \forall s \in S, \forall p \in P$, i.e. the optimal volume of each product (given in Fig. 1) that should be purchased from each supplier at each time period and the optimal volume of each product (given in Fig. 2) that should be stored in the inventory so that the total cost is minimum and the inventory level is as close as possible to the reference level decided by the decision maker. From Fig. 1, it can be seen that at time period 1, the manufacturer have to purchase 197 unit of product P1, 199 unit of product P2 and 3 unit of product P3 from supplier S1, 2 unit of product P1, 483 unit of product P2 and 109 unit of product P3 from supplier S2, and 298 unit of product P1 from supplier 3.

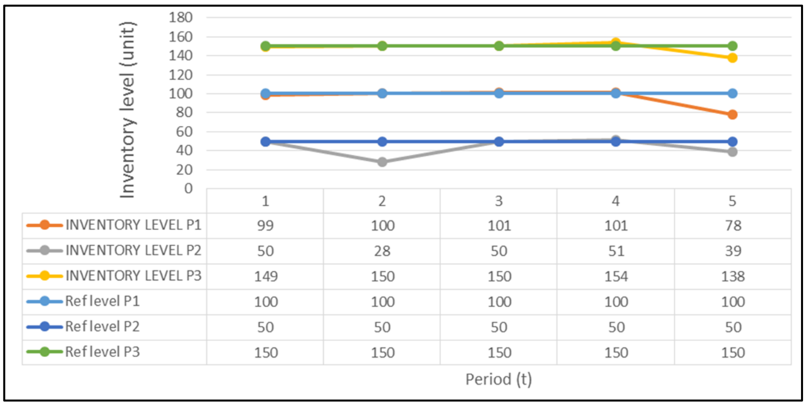

Figure 2. Inventory level and reference level in the first example

From Fig 2, it can be seen that at time period 1, the decision maker is deciding that the reference inventory level is 100 unit, 50 unit, and 150 unit of product P1, P2, and P3 respectively. The optimal strategy generated by the model is the manufacturer have to store 99 unit, 50 unit, and 149 unit of product P1, P2, and P3 respectively. The optimal decision for time period 2 to 5 can be derived analogously. The total cost for 5 time periods is 153398. From Fig. 2, it can be concluded that the inventory level is sufficiently closed to the reference level.

4.1.1. Continuous fuzzy demand

For the second example, let the demand $\tilde{D}_{tp}$ is a triangular fuzzy variable defined in Table 1 which can be illustrated by Fig. (3).

Table 1. Triangular membership function of $\tilde{D}_{tp}$

t	p	Triangular fuzzy membership function for $\tilde{D}_{tp}$ i.e. $\mu_{\tilde{D}_{tp}} = (a_{tp}, b_{tp}, c_{tp})$			Expected Value $E[\tilde{D}_{tp}]$
		a_{tp}	b_{tp}	c_{tp}	
1	P1	100	150	200	150
	P2	120	140	340	185
	P3	200	210	240	215
2	P1	180	210	220	205
	P2	160	200	210	192.5
	P3	120	140	160	140
3	P1	140	150	180	155
	P2	210	250	260	242.5
	P3	210	240	280	242.5
4	P1	240	260	285	261.25
	P2	180	220	260	220
	P3	190	210	240	212.5
5	P1	240	250	280	255
	P2	240	250	270	252.5
	P3	310	320	340	322.5

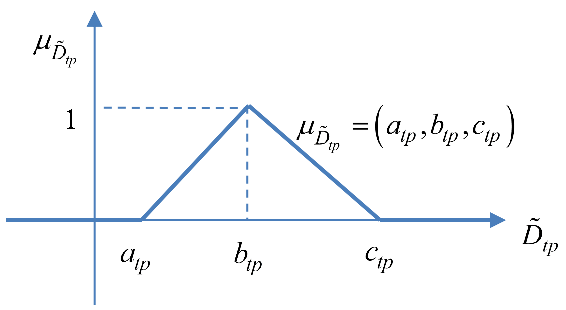

Figure 3. Triangular membership function $\mu_{\tilde{D}_{tp}} = (a_{tp}, b_{tp}, c_{tp})$

We evaluate the model for 5 time periods. The optimal strategy for this example is given in Fig. 4 and Fig. 5. From Fig. 4, it can be seen that at time period 1, 49 unit of product P1 have to be purchased from supplier S1, 210 unit of product P1 and 246 unit of product P2 have to be purchased from supplier S2. From Fig. 5, it can be seen that at time period 1, 100 unit of product P1, 50 unit of product P2, and 149 unit of product P3 have to be stored in the inventory that will be used to meet the demand at time period 2 or afterward. The optimal decision for time period 2 to 5 can be derived analogously. The total cost for 5 time periods is 94027. From Fig. 5, it can be conclude that the inventory level is sufficiently closed to the reference level.

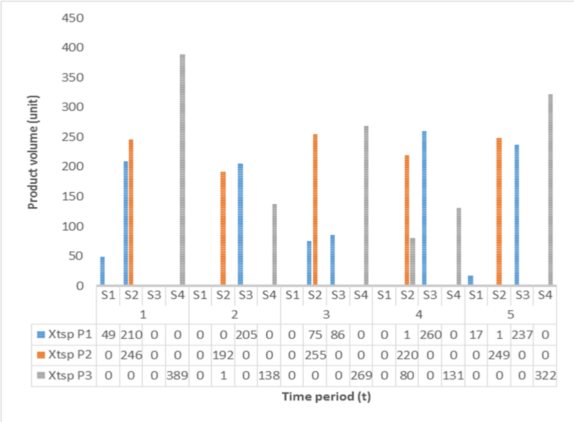

Figure 4. Optimal product volume that should be purchased from each supplier in the second example

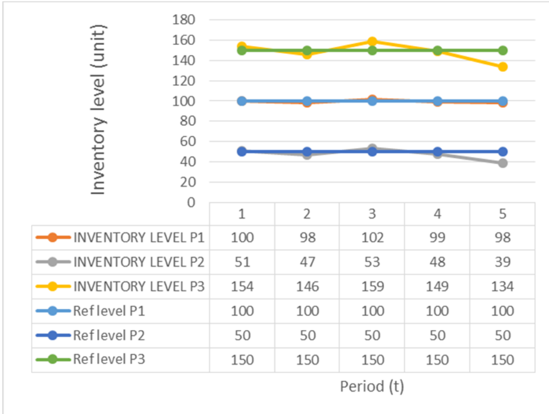

Figure 5. Inventory level and reference level in the second example

5. Conclusion and Future Works

In this paper, the optimal strategy determining for integrated inventory control and supplier selection problem with fuzzy demand was discussed. A mathematical model based on fuzzy expected value was considered to solve the problem. Two numerical examples was performed for model evaluating. From the results, it can be concluded that the problem was solved by the proposed model i.e. the optimal volume of each product that have to be purchased from each supplier for each time period and the optimal volume of each product for each time period that have to be stored were determined with minimum total cost and the inventory level was sufficiently closed to the reference level.

In the future works, we will develop the model so that it also have fuzzy cost parameters. Other methods like chance constrained and goal fuzzy programming, intuitionistic fuzzy programming and optimal control method will also be used to solve the problem and the comparison of them will also be discussed. Application in other problems will also discussed in the future articles.

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Appendix. Parameter values for numerical examples

Unit price (UPtsp)

Period	Supplier	Product		
		P1	P2	P3
All	S1	40	21	20
	S2	41	20	20
	S3	40	22	21
	S4	42	19	18

Transport cost (TCts)

period	S1	S2	S3	S4
all	1	2	1	2

Defect rate (Qtsp)

Period	Supplier	P1	P2	P3
all	S1	0.05	0.02	0.05
	S2	0.02	0.03	0.02
	S3	0.02	0.02	0.05
	S4	0.05	0.75	0.05

Late rate / Delay lead time rate (DLTtsp)

Period	Supplier	P1	P2	P3
all	S1	0.00	0.01	0.01
	S2	0.01	0.01	0.02
	S3	0.02	0.00	0.02

S4	0.01	0.02	0.00

Storage capacity (Mtp)			
period	P1	P2	P3
all	3000	3500	2500

Defect penalty cost per unit (UPCtsp)				
Period	Supplier	Product		
		P1	P2	P3
all	S1	0.2	0.4	0.25
	S2	0.5	0.6	0.5
	S3	0.75	0.2	0.7
	S4	0.5	1	0.25

Delay penalty cost per unit (UDCtsp)				
Period	Supplier	Product		
		P1	P2	P3
all	S1	0.2	0.2	0.4
	S2	0.5	0.2	0.2
	S3	0.1	0.1	0.1
	S4	0.5	0.5	0.2

Holding cost (HCtp)			
Period	Products		
	P1	P2	P3
all	0.1	0.1	0.2

Period	Total cost budget per period
all	300000

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