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Robust model predictive control for inventory system with uncertain demand using linear matrix inequalities

Robust model predictive control for inventory system with uncertain demand using linear matrix inequalities

Widowati[1, *)], R. Heru Tjahjana[2], Sutrisno[3], and Aditya Saputra[4]
[1, 2, 3, 4]Department of Mathematics, Faculty of Science and Mathematics, Diponegoro University, Jl. Prof Soedarto SH, Semarang 50275, Central Java, Indonesia.

[*)]corresponding author's e-mail: wiwied.mathundip@gmail.com, widowati_math@undip.ac.id

Abstract. In this paper, we develop an optimal control strategy on inventory systems with uncertain demand. To deal with these uncertainties we use a synthesis of robust model predictive control with linear matrix inequalities. The goal is to minimize the difference between the prediction and the reference trajectory subject to the objective function of each period, based on the input and output constraints. Using standard techniques, the optimization problem that minimizes the difference between the prediction and the reference trajectory, is reduced to a convex optimization problem involving linear matrix inequalities (LMIs). We provide numerical simulations on this system using MATLAB and then observe how robust predictive control models produce optimized strategy at the inventory level. In the simulation results, robust predictive control models provide an optimal strategy for controlling inventory levels with minimum total cost and inventory levels following inventory levels on issues.

Keywords : inventroy optimal strategy, uncertain demand, linear matrix inequalities, convex optimization.

1. Introduction

Predictive Control Method is one of the control strategies that are being developed in various sectors. Predictive control is a method based on optimization to control the system suitable to solve the problem of state stabilization and tracking. The main advantage of predictive control is its ability to handle input and state constraints. The reason that predictive control at any instant time of a set of future control signals is calculated by minimizing an objective function for a finite horizon considered in the current state. However, standard predictive control formulations do not directly consider the model of uncertainty and disturbance. Therefore, researchers continue to develop to solve the problem of uncertainty, one of which uses robust predictive control strategy.

In industrial issues, control strategies are often used to establish a policy that minimizes operational costs. There are many things that can be optimized in logistics and supply chain management but there are two most important operational costs of buying raw materials and storage costs [1, 2]. Inventories of raw materials in warehouses are used to meet future demand and for the purposes of inventory level control, inventory levels must be controlled with optimal strategies. The first thing to do is to formulate a mathematical model of inventory levels. Then, it can be controlled by using several control methods to determine the optimal strategy. The basic mathematical model of a product inventory system is proposed in [3] where the model is in a linear dynamic system and the optimal strategy is determined by a linear square control method. For single supply systems and multi-supplier supplies, the optimal strategy for controlling these inventories is presented by [4]. How to review supplies in the supply chain for multi-supplier cases is described in [5, 6] and this gives a mathematical model of the periodic review inventory system. The optimal policy for a one-period inventory system is considered in [7]. It was solved by assuming the cost of containment and lack of convex function. Another method of controlling an inventory system is the particle swarm optimization algorithm performed in [8]. Several studies have resulted in an optimal strategy for controlling inventory on many specified manufacturers, for example, the inventory policies in pharmaceutical systems and applicable to other manufactures with some adjustments [9]. Sutrisno and Wicaksono [10] proposed about optimal strategy for multi-product inventory system with supplier. In the other hand, predictive control for linear and hybrid system have been published by some researchers [11, 12, 13]. Further, for an uncertain demand supply, an optimal policy for inventory systems with highly uncertain demand have been published by Gallego, et.al [14]. Furthermore, Sutrisno, et.al [15] proposed an optimal strategy for inventory control problem where a demand and purchasing cost parameters are random.

In this paper, we perform an optimal strategy to control the inventory level of a single product inventory system with random demand so that the total cost will be minimized. The optimal strategy is calculated by using a robust predictive control model for inventory system with uncertain demand using linear matrix inequalities. We provide numerical simulations for this inventory system by using the model predictive control toolbox. Simulation results is given to propose how the model robust predictive control method generates the optimal strategies and how the dynamic of the stock level is compared to the desired level given by the decision maker.

2. Mathematical Modeling

This discussion describes the mathematical model of a single product inventory system with a single supplier and a random demand. The inventory model is presented in the form of a state space and then we determine an optimal control using Robust MPC algorithm. Suppose $y(k)$ is the inventory level of a product (goods) in the warehouse in a time period of observation *k*. The dynamics of supply y depend on goods coming from suppliers and demand *d*. Suppose that $u(k)$ is the number of goods coming from the supplier into the warehouse in the period *k*. Inventory dynamic $y(k)$ in the warehouse can be expressed as

$$\begin{aligned} y(k) &= u(0)+u(1)+u(2)+\cdots+u(k-L_p-1)-\beta_1 d-\beta_2 d-\beta_3 d\cdots-\beta_{k-1}d, \\ &= u(0)+u(1)+u(2)+\cdots+u(k-L_p-1)-(\beta_1 d+\beta_2 d+\beta_3 d\cdots+\beta_{k-1}d), \\ &= \sum_{j=0}^{k-L_p-1} u(j)-\sum_{j=1}^{k-1}\beta_j d, \end{aligned}$$

where d is demand that depends on uncertainty β . L_p is time delay on the delivery of goods from the supplier to the warehouse. The target inventory level is the number of demands in the first state variable denoted as $x_{d1}(k)=y_d(k)$ and $x_j(k)=u(k-L_p+j)$ for $j=2,3,\ldots,L_p+1$, then can be presented into the following system of state space system,

$$\left.\begin{aligned} x_1(k+1) &= x_1(k)+x_2(k)-\beta d(k) \\ x_2(k+1) &= x_3(k) \\ x_3(k+1) &= x_4(k) \\ &\vdots \\ x_{(n-1)}(k+1) &= x_n(k) \\ x_n(k+1) &= u(k) \end{aligned}\right\},$$

or can be presented into the state space system as follows:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ \vdots \\ x_{(n-1)}(k+1) \\ x_n(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ \vdots \\ x_{(n-1)}(k) \\ x_n(k) \end{bmatrix} + \begin{bmatrix} 0 & -\beta \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u(k) \\ d(k) \end{bmatrix},$$

$$y(k) = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ \vdots \\ x_{(n-1)}(k) \\ x_n(k) \end{bmatrix}.$$

The purpose of this problem is to minimize the total cost and to know the inventory level y approaching the desired inventory level, so as to minimize the value between y and y_d by using the following objective function,

$$\min_u J(x,u) = \sum_{j=0}^{k-1} \Big(\left[y(j) - y_d \right]^T R \left[y(j) - y_d \right] + Qu(j) \Big),$$

with constraints

$$0 \le u_k \le S_c,$$

$$0 \le y_k \le W_c,$$

where S_c is maximum capacity of suppliers and W_c is the maximum capacity of the warehouse.

3. Linear Matrix Inequalities

The problem of optimal control on systems with uncertainty structures bounded by boundary norms can be solved using LMI (Linear Matrix Inequalities) approach. Furher, we proposed about the relationship between control theory and LMI. The formulation of the problem no longer leads to quadratic problems, but rather leads to the LMI optimization problem which is a convex problem. The following is explained about LMI. Linear Matrix Inequality is an inequality matrix with form

$$F(u) = F_0 + \sum_{i=1}^{l} v_i F_i \ge 0,$$

with F_i is a symmetric matrix, and v_i is the scalar variable appears in affine inequality. It is clear that one set of LMIs, $F_1(v) \ge 0, \ldots, F_m(v) \ge 0$ equal to one LMI, is obtained by arranging all matrices into one block-diagonal matrix. The most important thing in the LMI is the type of optimization problem, i.e. LMI, $F(v) \ge 0$ that is a convex problem. Many convex problems can be written in this form [9], and an efficient algorithm to solve such problems. Specifically, if, $Q(v) = Q(v)^T$; $R(v) = R(v)^T$ and $Q(v), R(v), S(v)$ depend affinely on *v*, then the inequalities

$$Q(v) > 0,$$

$$R(v) - S(v)^T Q(v)^{-1} S(v) \ge 0,$$

is equivalent to LMI

$$\begin{bmatrix} Q(v) & S(v) \\ S(v)^T & R(v) \end{bmatrix} \geq 0.$$

Let, the quadratic programming convex problem

$$\min_x x^T Qx + q^T x + r, \text{ subject to}$$

$$Ax \geq b$$

.

is equivalent to

$$\min_{\gamma,x} \gamma \text{ subject to } \begin{cases} \gamma - x^T Qx - q^T - r \geq 0, \\ Ax \geq b, \end{cases}$$

which is equivalent to

$$\min_{\gamma,x} \gamma \text{ subject to}$$

$$\begin{bmatrix} I & Q^{1/2}x \\ x^T Q^{1/2} & \gamma - q^T x - r \end{bmatrix} \geq 0 \quad \text{and} \quad \begin{bmatrix} (Ax-b)_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & (Ax-b)_m \end{bmatrix} \geq 0.$$

4. Numerical Simulation

The inventory system used is a single product inventory system and a single supplier with time delivery of goods from time of order is $L_p = 2$, and in this case the equation system is given uncertainty β, with $\beta \in [0,1]$. Then it can be written in polytope form as follows,

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B_0 = \begin{bmatrix} 0 & -1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & -0.5 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 0 & -0.75 \\ 0 & 0 \\ 1 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},\ D = [0 \quad 0],$$

which can be expressed in the polytope matrix

$$S(k) \in \mathrm{Co}\{S_1,\ S_2,\ S_3, S_4\} = \left\{ \sum_{i=1}^{4} \alpha_i S_i : \alpha_i \geq 0, \sum_{i=1}^{4} \alpha_i = 1 \right\}$$

$$S_1 = \begin{bmatrix} A & B_0 \\ C & D \end{bmatrix},\ S_2 = \begin{bmatrix} A & B_1 \\ C & D \end{bmatrix},\ S_3 = \begin{bmatrix} A & B_2 \\ C & D \end{bmatrix}, S_4 = \begin{bmatrix} A & B_3 \\ C & D \end{bmatrix}.$$

The above system is a real system used to obtain an inventory system with random demand. While to predict the nominal system used as follows,

$$x(k+1) = A_o x(k) + B_o u(k),$$
$$y(k) = Cx(k) + Du(k),$$

where

$$A_0 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B_0 = \begin{bmatrix} 0 & -1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},\ D = [0 \quad 0].$$

Based on the calculation using MATLAB it is known that the controlability matrix (A_0, B_0) has full-row rank that is 3. In addition, the observability matrix (C, A_0) has full colum rank that is 3. So it can

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be concluded that the system used can be controlled and observed. Then the system is applied in the inventory problem to obtain optimal output that is the optimal inventory level in the warehouse.

Given the following inventory issues: suppose that the cost of re-supply the product in the shipment per unit of product is 100 and the storage cost per product unit is 30. The supplier may ship items up to 2000 units per period of the appointment and the storage capacity is 5000 units. Based on the given problem, we get the mathematical model formulation of objective function as follows

$$\min_{u} J(x,u) = \sum_{j=0}^{k-1} \left(\left[y_k - y_d \right]^T 30 \left[y_k - y_d \right] + 100 u_k \right),$$

with constraints,

$$0 \le u_k \le 2000,$$

$$0 \le y_k \le 5000.$$

Calculation to determine optimal control or in other words optimal order quantity to minimize operational cost is done using MPC Controller toolbox in MATLAB 2015b. the calculation is done by supposing the inventory level at the beginning of the period is 0, and the demand has the Gaussian distribution with the initial condition is 1000 units.

The simulation performed illustrates the two system inputs shown in Figure 3.1, the first being the product level arriving at the optimal warehouse (shipment) of 1150 units at the beginning of the observation period and stable afterwards at the level of 1000 units until the end of the observation period. Second, it is described the spread of demand uncertainty according to the Gaussian probability distribution with an initial value of 1000 units.

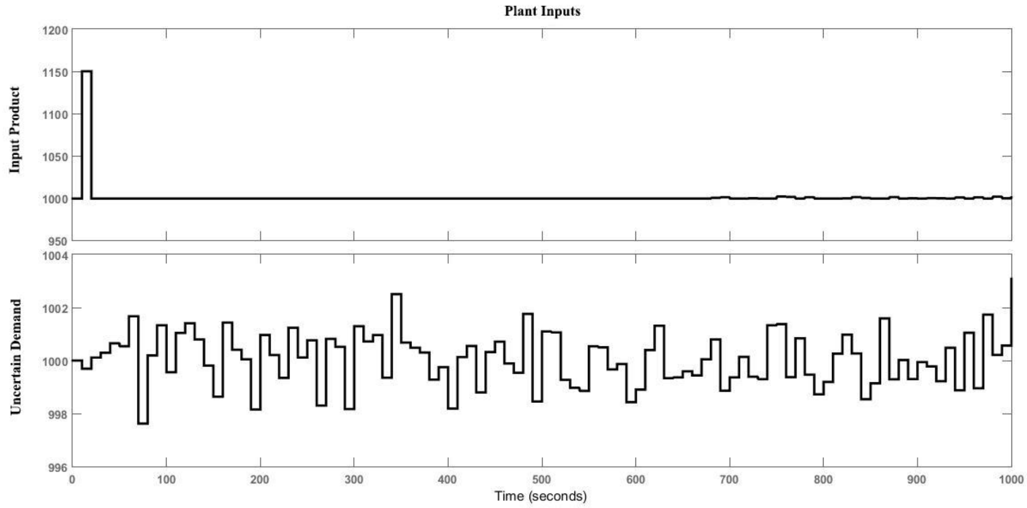

Figure 4.1 Optimal control of the inventory system

From Figure 3.2 is shown that the output has an optimal value, it indicates that the optimal inventory level in the warehouse that is stable in the setpoint circle is 4000 units.

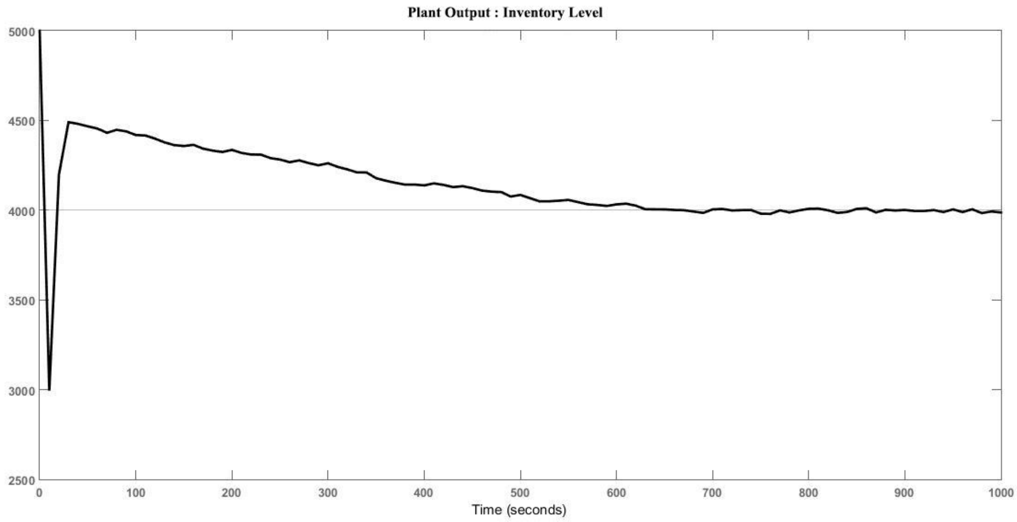

Figure 4.2 The output: inventory level of the product

The optimal inventory level is obtained from two system inputs namely the level of product input into the warehouse (shipment) and the number of orders from consumers that the number is uncertain. In accordance with the simulation results using MATLAB, it can be concluded that the level of supply of products to the warehouse of 1150 units of products in the early period and 1000 units of products each period further up to the 30th observation period, with the level of product inventory in the warehouse of 4000 units. Due to the cost to re-supply of 100 per unit of product and storage cost of 30 per unit of product, the estimated minimum purchase and storage costs corresponding to (3.147) to period $k = 30$ can be calculated as follows

$$
\begin{aligned}
O_c &= \sum_{j=0}^{29}(P_c + H_c)u_j = \sum_{k=0}^{29} Q_c u_j, \\
&= (100+30)\times 1150 + \sum_{j=1}^{29}(100+30)\times 1000, \\
&= (130\times 1150) + (130\times 29\times 1000), \\
&= 150000 + 3770000, \\
&= 3920000.
\end{aligned}
$$

Thus, the cost of purchase (shipment) and product storage for 30 periods amounted to 3.92 million.

5. Concluding Remarks

In this work, the robust model predictive control was used to determine the optimal strategy for controlling the single product multi-period inventory system with uncertain demand. The formulated dynamics of this system was presented the dynamic of the stock level of the product in this inventory system. A numerical simulation was performed in model predictive toolbox in MATLAB. From the simulation results, we find that the stock level for the product approached the desired level well.

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