Optimal Strategy Analysis Based on Robust Predictive Control for Inventory System with Random Demand

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Abstract. In this paper, the optimal strategy for a single product single supplier inventory system with random demand is analyzed by using robust predictive control with additive random parameter. We formulate the dynamical system of this system as a linear state space with additive random parameter. To determine and analyze the optimal strategy for the given inventory system, we use robust predictive control approach which gives the optimal strategy i.e. the optimal product volume that should be purchased from the supplier for each time period so that the expected cost is minimal. A numerical simulation is performed with some generated random inventory data. We simulate in MATLAB software where the inventory level must be controlled as close as possible to a set point decided by us. From the results, robust predictive control model provides the optimal strategy i.e. the optimal product volume that should be purchased and the inventory level was followed the given set point.

INTRODUCTION

Control system is one of the most important components in the manufacturing industry. To increase the profit, manufacturers must optimize their operational costs. There are many things that can be optimized in logistics and supply chain management but there are two most important operational costs which are purchasing raw material cost and storage cost [1]. Inventory of raw material in warehouse is used to meet the future demand and for inventory level control purposes, hence the inventory level must be controlled with optimal strategy. The first thing to do is to formulate the mathematical model of the inventory level. Then, it can be controlled by using several control methods to determine its optimal strategy. The basic mathematical model of a single product inventory system was presented in [2] where the model was in linear dynamical system and the optimal strategy was determined by linear quadratic control method. For single product and multi-supplier inventory system, the optimal strategy for controlling this inventory was considered in [3]. How to review the inventory in supply chain for multi-supplier case was explained in [4] and it gives the mathematical model of periodic review inventory system. The optimal policy for single-period inventory system was considered in [5]. It was solved by assuming the holding cost and shortage cost function are convex. Another method to control an inventory system is particle swarm optimization algorithm that was performed in [6]. Some researches were generate the optimal strategy to control the inventory on many specific manufactures, for example, reference [7] performed the inventory policies in pharmaceutical system and it can be applied to other manufactures with some adjustments. For inventory uncertain demand, reference [8] gave the optimal policies for inventory system under highly uncertain demand. In control system, the basic model predictive control (MPC) is an optimization based method for controlling the linear discrete systems and it is applicable for state stabilization and state tracking problem [9]. Moreover, MPC can be applied for many kind of dynamic systems, for example, MPC can be applied to control linear systems and nonlinear systems. Lately, it can be applied to control a hybrid systems [10]. MPC is the famous for practitioners or researchers and it is applied to solve many control problems. MPC is performed by predicting the state and input variables along prediction horizon, substituting these predictions into objective function and solving it using optimization method to determine the optimal control action. Recently, works using model predictive control (MPC) had been found to provide the attractive alternative for inventory control [11] and supply chain management [12]. These approaches are conceptually different and require less detailed knowledge in comparison with cost-optimal stochastic programming solutions that require many what-if cases to be ran and examined by highly skilled professionals [13] . MPC offers the same flexibility in terms of the information sharing,

International Conference and Workshop on Mathematical Analysis and its Applications (ICWOMAA 2017)
AIP Conf. Proc. 1913, 020017-1–020017-5; https://doi.org/10.1063/1.5016651
Published by AIP Publishing. 978-0-7354-1605-5/$30.00

network topology, and constraints that can be handled. The appeal of MPC for dynamic inventory management in supply chain can be summarized as follows: as an optimizer, MPC can minimize or maximize an objective function that represents a suitable measure for supply chain performance. As a controller, MPC can be tuned to achieve stability, robustness, and performance in the presence of plant-model mismatch, failures, and disturbances that affect the system.

In this paper, we will perform the optimal strategies to control the inventory level of single-product inventory system with random demand so that the total cost will be minimized. The optimal strategies will be computed by using robust model predictive control after the dynamic of this system is modeled. We will give the numerical simulation for this inventory system by using model predictive control toolbox in MATLAB. From simulation results, we will observe how the model predictive control method generates the optimal strategies and how the dynamic of the stock level is compared to the desired level given by the decision maker

MATHEMATICAL MODEL

The inventory system that will be discussed is a single product inventory system with single supplier and uncertain demand where the uncertainty of the demand will be assumed as a random variable with some probability distribution. Suppose $y(k)$is the inventory level of a product (goods) in the warehouse in a time period of observation k. The dynamics of supply depends on the goods coming from the supplier and demand d. In this article, we have assumptions that demand d is random variable and the inventory level before the observation is 0i.e.for $k < 0,\ y(k) = 0$. Suppose that $u(k)$ is the number of goods coming from the supplier into the warehouse in the period k. the basic model of this system without random demand was provided in [2]. Inventory dynamic $y(k)$ in the warehouse can be expressed as

$$\begin{aligned} y(k) &= u(0) + u(1) + u(2) + \cdots + u(k - L_p - 1) - d(1) - d(2) - \cdots - d(k-1) \\ &= \sum_{j=0}^{k-L_p-1} u(j) - \sum_{j=0}^{k-1} d(j) \end{aligned} \tag{1}$$

where L_p is the lead time in the delivery of goods from supplier to the warehouse.The target inventory level is the number of demands in the first state variable denoted as $x_{d1}(k) = y_d(k)$and $x_j(k) = u(k - L_p + j)$ for $j = 2, 3, \ldots, L_p + 1$, then (1) can be presented into the following system of state space system :

$$\begin{aligned} x(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\begin{bmatrix} u(k) \\ d(k) \end{bmatrix} \\ y(k) &= \mathbf{C}'\mathbf{x}(k) \end{aligned} \tag{2}$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}.$$

Hence, system(2) can be rewritten as

$$
\begin{aligned}
x(k+1) &= \begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ \vdots \\ x_{(n-1)}(k) \\ x_n(k) \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u(k) \\ d(k) \end{bmatrix} \\
y(k) &= \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ \vdots \\ x_{(n-1)}(k) \\ x_n(k) \end{bmatrix}.
\end{aligned}
\tag{3}
$$

Inventory system represented by system(2)will be controlled so that the system output i.e.the inventory level y will be approaching to the desired level of inventory y_d as close as possible. It is assumed that the cost component consists of the purchase cost per unit of the product from the supplier and the storage cost per unit product. Let's say the purchase cost per product unit P_c and storage cost per product unit H_c so that the total cost per product unit O_c per period $k = 0, 1, 2, 3, ..., k-1$ is

$$
O_c = \sum_{j=0}^{k-1} P_c u(j) + \sum_{j=0}^{k-1} H_c u(j) = \sum_{j=0}^{k-1} Q_c u(k) \tag{4}
$$

with $Q_c = P_c + H_c$.

NUMERICAL SIMULATION

The robust model predictive control that we used to be applied to control the inventory system given in the previous section is following the robust MPC given by [9]. Consider plant (2) where d is white noise with covariance matrix $E\left(dd^T\right) = W \geqslant 0$, and d supposed to be the disturbance. By Applying Kalman filter, the optimal filtered state estimate $\hat{x}$ can be written as

$$
\begin{aligned}
\hat{x}(t+1) &= A\hat{x}(t) + Bu(t) \\
\hat{x}(t+l) &= A\hat{x}(x+l-1) + B\hat{u}(k+l-1) \text{ for } l > 1.
\end{aligned}
\tag{5}
$$

By predicting the future state, input and output and substituting the prediction vectors into objective function

$$
J(t) = \sum_{k=0}^{N_y} \left(y(t+k) - y_{ref}\right)^T Q_k \left(y(t+k) - y_{ref}\right)^T \sum_{k=0}^{N_u} u(t+k)^T R_k u(t+k) \tag{6}
$$

where $Q = blockdiag\{Q_i\}_{i=0,1,...,N_y}, R = blockdiag\{R_i\}_{i=0,1,...,N_u}$, then the optimal input can be obtained by minimizing J by using quadratic programming. To perform this control method, we use model predictive control toolbox in MATLAB and simulating (2) with the following generated data. Suppose the purchase cost of the product is 1000 per unit and the storage cost in the warehouse is 100 per unit. Supplier can deliver goods up to 20000 units. The maximum warehouse capacity is 50000 units.Let the initial inventory level of the product is 5000 units and the demand is following the Gaussian distribution with initial condition 100 unit. The simulation results are given in Fig. 1 - 2.

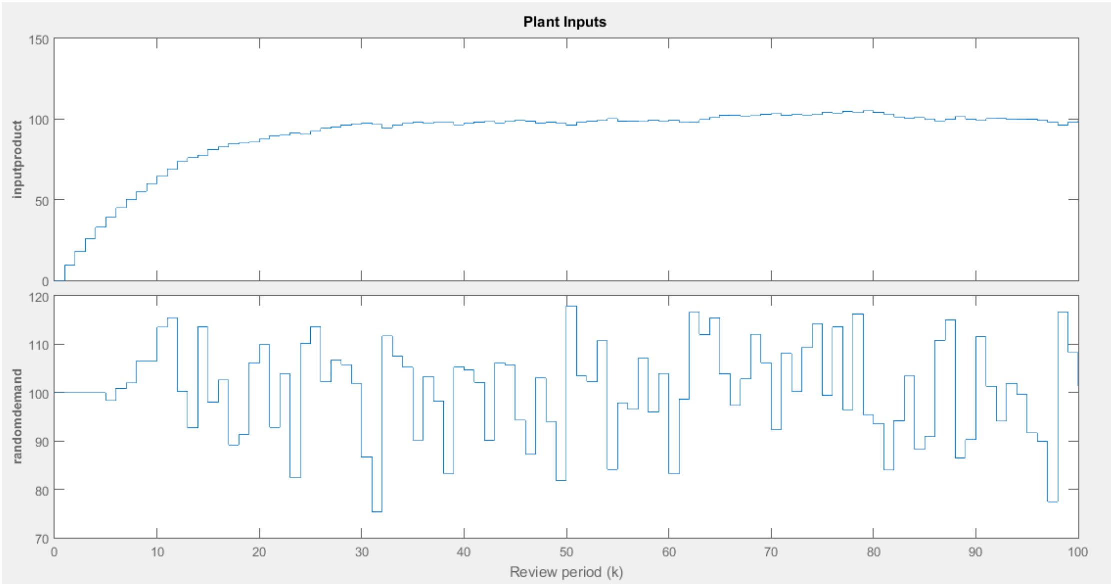

FIGURE 1. Optimal Input (shipment) $u(k)$ and simulated random demand $d(k)$

Fig.1 shows the optimal input or optimal strategy generated by robust MPC. If this optimal input or optimal strategy showed by Fig.1 is applied to this inventory system, it will affects the output that is the dynamic of the stock level for the product given by Fig.2.

Fig. 2 shows dynamic of the stock level for the product. The initial value of stock level of product is 5000 items, after met the demand and received the product from supplier for day 1 to 30, it was sufficiently closed to the desired level i.e. about 3800 items.

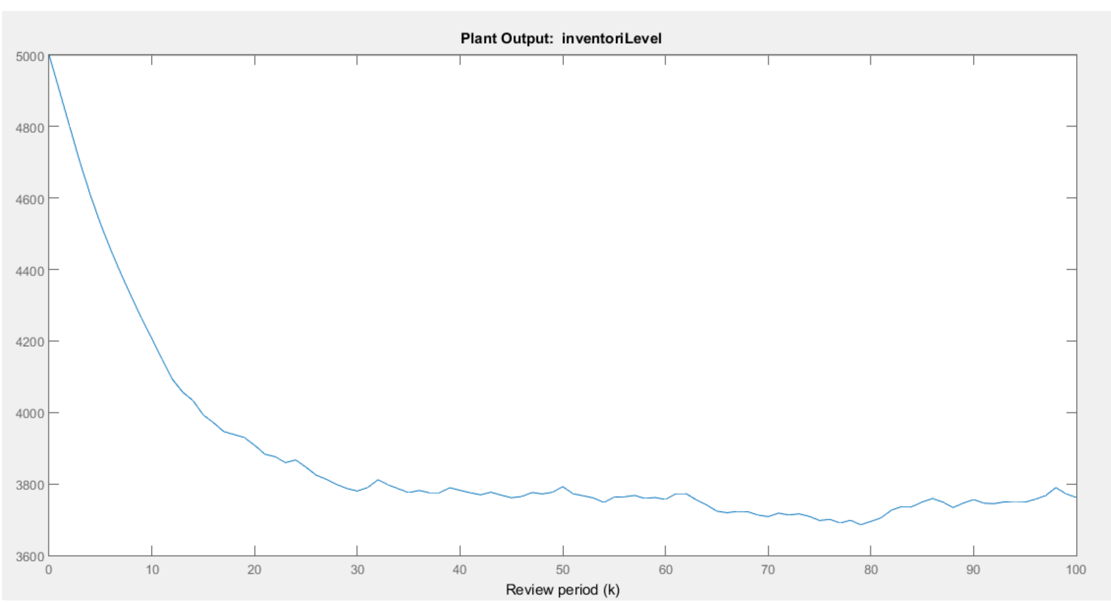

FIGURE 2. The output: inventory level of the product

CONCLUDING REMARKS

In this paper, the robust model predictive control was used to determine the optimal strategy for controlling the single product multi-period inventory system with uncertain demand. The formulated dynamics of this system was presented the dynamic of the stock level of the product in this inventory system. A numerical simulation was performed in model predictive toolbox in MATLAB. From the simulation results, it can be conclude that the stock level for the product approached the desired level well.

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