

PAPER • OPEN ACCESS

Dynamic optimization approach for integrated supplier selection and tracking control of single product inventory system with product discount

To cite this article: Sutrisno *et al* 2017 *IOP Conf. Ser.: Mater. Sci. Eng.* **166** 012028

View the [article online](#) for updates and enhancements.

Related content

- [Expected value based fuzzy programming approach to solve integrated supplier selection and inventory control problem with fuzzy demand](#)
Sutrisno, Widowati, Sunarsih et al.
- [Optimal strategy for supplier selection problem integrated with optimal control problem of single product inventory system with piecewise holding cost](#)
Sutrisno, Widowati and R H Tjahjana
- [A parallel optimization method for product configuration and supplier selection based on interval](#)
Jian Zheng, Meng Zhang and Guoxi Li

Dynamic optimization approach for integrated supplier selection and tracking control of single product inventory system with product discount

Sutrisno¹, Widowati² and R. Heru Tjahjana³

Department of Mathematics, Diponegoro University, Jl. Prof. Soedarto, SH,
Tembalang, Semarang, Indonesia, 50275

Emails: ¹tresno.math@undip.ac.id, ²widowati_math@undip.ac.id,
³heru_tjahjana@undip.ac.id

Abstract. In this paper, we propose a mathematical model in the form of dynamic/multi-stage optimization to solve an integrated supplier selection problem and tracking control problem of single product inventory system with product discount. The product discount will be stated as a piece-wise linear function. We use dynamic programming to solve this proposed optimization to determine the optimal supplier and the optimal product volume that will be purchased from the optimal supplier for each time period so that the inventory level tracks a reference trajectory given by decision maker with minimal total cost. We give a numerical experiment to evaluate the proposed model. From the result, the optimal supplier was determined for each time period and the inventory level follows the given reference well.

1. Introduction

Logistics & Supply Chain Management (LSCM) involves the management of many components that are procurement, movement and storage of materials, parts and finished product [1]. Supplier selection and inventory control system are two important components in LSCM. Supplier selection problem is appeared when there are several supplier alternatives that have to be determined who the optimal supplier(s) is. In other hand, manufacturer has to decide when and how many he has to store the product in his inventory in order to satisfy the demand with minimal cost. Some manufacturer decides to control his inventory level so that the stock level of the product is located at some point as close as possible to a reference point or trajectory given by the decision maker.

To find the optimal strategy so that the total cost is minimal, several mathematical models were developed by many researchers. We can classify the model into single product or multi product model. For single product model, [2, 3, 4] were determined the optimal strategy for inventory controlling by without solving the supplier selection problem. In the other hand, [5, 6, 7] were solved the dynamic supplier selection problem, but the inventory of the product was not controlled. By applying some assumption, several researchers were developed the model for integrated supplier selection and inventory control problem. Reference [8] was developed a model for integrated supplier selection problem and inventory system by assuming that the decision maker was not decided to control the inventory level to follows some reference point. In the other hand, [9] was integrated the production control with multi supplier without inventory level controlling and [10] was integrated the supplier selection problem and optimal control problem of a single product by assuming that there is no product discount on the purchasing cost.



In this paper, we formulate a mathematical model in the dynamic/multi-stage optimization form to determine the optimal strategy for an integrated supplier selection problem and trajectory tracking control problem of an inventory system with product discount. The dynamic programming will be used to solve this optimization. A numerical experiment will be performed to evaluate and simulate the model and to illustrate how the problem will be solved.

2. Mathematical Model

Suppose that a manufacturer will purchase a product from S suppliers with different purchase cost per unit. Let $X_{t,s}$ denotes the product volume purchased from supplier s at time period t and $U_{t,s}$ denotes the purchasing cost per unit from supplier $s \in S$ at time period $t \in T$ where S denotes the set of the suppliers and T denotes the set of time period. Assume that there is product discount from supplier s with the scheme as follows

$$U_{t,s} = \begin{cases} U_{t,s}^{(1)}, & \text{if } 0 = d_s^{(0)} < X_{t,s} \leq d_s^{(1)} \\ U_{t,s}^{(2)}, & \text{if } d_s^{(1)} < X_{t,s} \leq d_s^{(2)} \\ \vdots \\ U_{t,s}^{(J)}, & \text{if } d_s^{(J-1)} < X_{t,s} \leq d_s^{(J)} \end{cases}, \forall t \in T, \forall s \in S \tag{1}$$

or

$$U_{t,s} = U_{t,s}^{(j)}, \text{ if } d_s^{(j-1)} < X_{t,s} \leq d_s^{(j)}, \forall t \in T, \forall s \in S \tag{2}$$

where $d_s^{(j)}, j = 0, 1, 2, \dots, J$ is a constant number decided by the supplier s . This product discount scheme can also be called as price level. Let H_t denotes the holding cost per unit at time period t , D_t denotes the demand of the product at time period t and I_t denotes the inventory/stock level of the product at time period t . The decision maker decides that the inventory level will be controlled so that it will be located at some point as close as possible to a reference point or trajectory given by the decision maker. Let r_t denotes the reference point at time period t , we define $(I_t - r_t)^2$ as the track reference objective function. Assume that the cost components in this problem are purchasing cost and holding cost, then we define the following optimization which is minimizing the total cost i.e.

$$\min \begin{cases} \sum_{t=1}^T \sum_{s=1}^S U_{t,s}^{(1)} X_{t,s} + \sum_{t=1}^T H_t I_t + \sum_{t=1}^T (I_t - r_t)^2, & \text{if } 0 = d_s^{(0)} < X_{t,s} \leq d_s^{(1)} \\ \sum_{t=1}^T \sum_{s=1}^S U_{t,s}^{(2)} X_{t,s} + \sum_{t=1}^T H_t I_t + \sum_{t=1}^T (I_t - r_t)^2, & \text{if } d_s^{(1)} < X_{t,s} \leq d_s^{(2)} \\ \vdots \\ \sum_{t=1}^T \sum_{s=1}^S U_{t,s}^{(J)} X_{t,s} + \sum_{t=1}^T H_t I_t + \sum_{t=1}^T (I_t - r_t)^2, & \text{if } d_s^{(J-1)} < X_{t,s} \leq d_s^{(J)} \end{cases} \tag{3}$$

or

$$\min \left\{ \sum_{t=1}^T \sum_{s=1}^S U_{t,s}^{(j)} X_{t,s} + \sum_{t=1}^T H_t I_t + \sum_{t=1}^T (I_t - r_t)^2, \text{ if } d_s^{(j-1)} < X_{t,s} \leq d_s^{(j)}, j = 1, 2, \dots, J. \right. \tag{4}$$

Next we formulate the constraints for this optimization model. To ensure that the demand of the product will be satisfied at each time period, we give the first constraint as follows

$$I_{t-1} + \sum_{s=1}^S X_{t,s} - I_t \geq D_t, \forall t \in T. \quad (5)$$

If the supplier s has maximum capacity C_s to supply the product for any time period t , then we have the second constraint as follows

$$X_{t,s} \leq C_s, \forall t \in T, \forall s \in S. \quad (6)$$

For any time period, if the inventory has maximum capacity M , then we have the third constraint as follows

$$I_t \leq M, \forall t \in T. \quad (7)$$

Integer constraint for the purchased product volume can be written as follows

$$X_{t,s} \in \{0, 1, 2, \dots\}, \forall t \in T, \forall s \in S. \quad (8)$$

3. Dynamic Programming

Dynamic programming or multi-stage programming is a technique that can be used to solve an optimization problem by working backward (or forward) iteration from the end of a problem toward the beginning of the problem [11]. Dynamic programming divides the problem into stages where each stage has several associate states and uses the recursion function to do backward or forward iteration. There are two types of the dynamic programming which are deterministic when all of parameters are known with certainty or stochastic where at least one of the parameters uncertain or random. Dynamic programming can be used to solve many optimization problem classes such as dynamic linear programming, dynamic quadratic programming, dynamic nonlinear programming, dynamic integer programming and mixed integer programming.

Several optimization tools were developed that can be used to perform a dynamic programming. In this paper, we use LINGO 15.0 given in [12] to solve the optimization problem (3) where the model class is pure integer quadratic programming (PIQP).

4. Numerical Experiment

Suppose that a manufacture has four suppliers which are s_1 , s_2 , s_3 and s_4 . For any time period, the purchasing cost from s_1 is \$12/unit if the manufacturer purchasing less or equal to 100 units and it will be \$9/unit if the manufacturer purchasing the product for more than 100 units but no more than 120 units since the maximum capacity of supplier s_1 is 120 units. The purchasing cost from supplier s_2 is \$13/unit if the manufacturer purchasing less or equal to 150 units and it will be \$9/unit if the manufacturer purchasing more than 150 units but no more than 200 units since the maximum capacity of supplier s_2 is 200 units. The purchasing cost from supplier s_3 is \$13/unit if the manufacturer purchasing less or equal to 100 units and it will be \$9/unit if the manufacturer purchasing more than 100 units but no more than 200 units since the maximum capacity of supplier s_3 is 200 units. Finally, the purchasing cost from supplier s_4 is \$14/unit if the manufacturer purchasing less or equal to 120 units and it will be \$8/unit if the manufacturer purchasing more than 120 units but no more than 150 units since the maximum capacity of supplier s_4 is 150 units. The purchasing cost per product unit from all suppliers is summarized by Table (1) and the demand of the product for time period 1 to 12 is given by Table (2).

Suppose that the initial inventory level is 0 item and the holding cost is \$1/unit/period. Finally, the warehouse's maximum capacity is 200 units/period. The decision maker desires that the inventory/stock level must be located at some point as close as possible to the reference level 100 units with minimal cost.

Table 1. Purchasing cost & supplier capacity

Supplier	Purchasing cost	Supplier capacity (units/period)
s_1	$\begin{cases} \$12 & \text{if } X_{t,1} \leq 100 \\ \$9 & \text{if } X_{t,1} > 100 \end{cases}$	120
s_2	$\begin{cases} \$13 & \text{if } X_{t,2} \leq 150 \\ \$10 & \text{if } X_{t,2} > 150 \end{cases}$	200
s_3	$\begin{cases} \$13 & \text{if } X_{t,2} \leq 100 \\ \$9 & \text{if } X_{t,2} > 100 \end{cases}$	200
s_4	$\begin{cases} \$14 & \text{if } X_{t,2} \leq 120 \\ \$8 & \text{if } X_{t,2} > 120 \end{cases}$	150

Table 2. Demand of the product

Time period (t)	Demand (unit)
1	150
2	70
3	80
4	120
5	140
6	100
7	320
8	250
9	270
10	180
11	310
12	270

We solve optimization problem (4) for 12 time periods with 3-by-3 of time periods by using LINGO 15.0 in Windows 8 AMD A6 2.7GHz of processor and 4 GB of Memory. The optimal strategy for this problem is summarized in Table 3.

Table 3. Solution of the problem

Time period (t)	$X_{t,s}$				Demand (unit)	Inventory (unit)
	s_1	s_2	s_3	s_4		
1	101	0	0	150	150	101
2	68	0	0	0	70	99
3	75	0	0	0	80	94
4	0	0	0	125	120	99
5	0	0	0	138	140	97
6	0	0	101	0	100	98
7	0	0	171	150	320	99
8	0	0	101	150	250	100
9	115	0	0	150	270	95
10	33	0	0	150	180	98
11	0	0	162	150	310	100
12	115	0	0	150	270	95

Table 3 shows the optimal strategy for time period 1 to 12. It can be seen that at time period 1, the manufacturer has to purchase 101 units from supplier s_1 and 150 units from supplier s_4 while s_2 and s_3 was not selected to supply the product. At time period 2, the manufacturer has to purchase only 68 units from supplier s_1 . The evolution of the inventory level of the product and its reference point can be seen in Fig. 1.

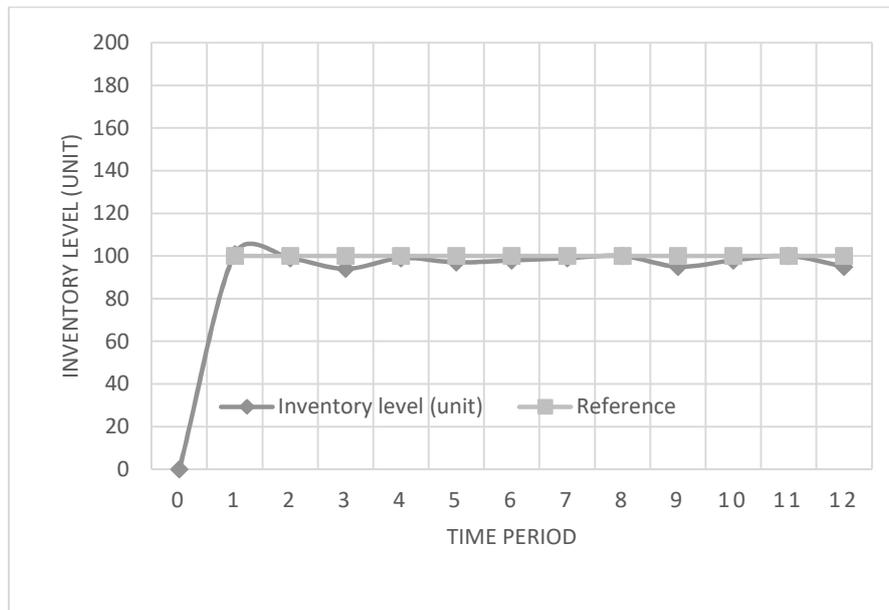


Figure 1. The evolution of the inventory level of the product and its reference trajectory

From Fig. 1, it can be seen that the initial inventory level is 0 unit and the reference inventory is 100 unit for all time period. At time period 1 to 12, the inventory level is sufficiently closed to the reference point. The largest deviation between the actual inventory level and the reference inventory is 6 units at time period 3 and the smallest deviation is 0 unit at time periods 8 and 11. It can be concluded that the inventory level follows the reference point well.

5. Conclusion and Future Works

In this paper, a mathematical model in the form of dynamic/multi-stage optimization was considered to solve an integrated supplier selection problem and tracking control problem of single product inventory system with product discount. The optimal strategy was determined by using dynamic programming. Numerical experiment was considered and from the result, it can be conclude that the supplier selection problem was solved and the inventory/stock level followed the reference point well.

In other work, we propose the mathematical model in the form of stochastic dynamic optimization to solve the integrated supplier selection problem and trajectory tracking control problem for inventory system with product discount in probabilistic environment. In the future works, we will develop the model for multi-product inventory system including order cost, service level parameter, late delivery parameter, etc. We will also develop the mathematical model with carrier selection problem solving.

Acknowledgment

Authors would like to thank Universitas Diponegoro for funding support under Professorship Research Program, PNBP DIPA Universitas Diponegoro 2016.

References

- [1] M. Christopher, *Logistics & Supply Chain Management*, Great Britain: Pearson Education Limited, 2011.
- [2] Y. Arda and J.-C. Hennet, "Inventory control in a multi-supplier system," *Int. J. Production Economics*, vol. 104, pp. 249-259, 2006.

- [3] W. Guerrero, T. Yeung and C. Guéret, "Joint-optimization of inventory policies on a multi-product multi-echelon pharmaceutical system with batching and ordering constraints," *European Journal of Operational Research*, vol. 231, pp. 98-108, 2013.
- [4] S. Minner, "Multiple-supplier inventory models in supply chain management: A review," *Int. J. Production Economics*, Vols. 81-82, pp. 265-279, 2003.
- [5] N. R. Ware, S. P. Singh and D. K. Banwet, "A mixed-integer non-linear program to model dynamic supplier Selection Problem," *Expert Systems with Applications*, vol. 41, pp. 671-678, 2014.
- [6] C. Haksever and J. Moussourakis, "A model for optimizing multi-product inventory systems with multiple constraints," *International Journal of Production Economics*, vol. 97, pp. 18-30, 2005.
- [7] S. S. Kara, "Supplier selection with an integrated methodology in unknown environment," *Expert Systems with Applications*, vol. 38, pp. 2133-2139, 2011.
- [8] C. Guo and L. Xueping, "A multi-echelon inventory system with supplier selection and order allocation under stochastic demand," *Int. J. Production Economics*, vol. 151, pp. 37-47, 2014.
- [9] A. Hajji, A. Gharbi, J.-P. Kenne and R. Pellerin, "Production control and replenishment strategy with multiple suppliers," *European Journal of Operational Research*, vol. 208, pp. 67-74, 2011.
- [10] Sutrisno and P. A. Wicaksono, "Optimal Strategy for Multi-product Inventory System with Supplier Selection by Using Model Predictive Control," in *Procedia Manufacturing of Industrial Engineering and Service Science (IESS)*, Yogyakarta, 2015.
- [11] W. L. Winston, *Operations Research: Applications and Algorithms*, 2004.
- [12] L. S. Team, *LINGO: The Modeling Language and Optimizer*, Illinois: Lindo Systems Inc., 2014.
- [13] S. M. Mousavi, S. Niaki, A. Bahreininejad and S. N. Musa, "Multi-Item Multiperiodic Inventory Control Problem with Variable Demand and Discounts: A Particle Swarm Optimization Algorithm," *The Scientific World Journal*, vol. 2014, pp. 1-16, 2014.