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Stability Analysis and Control of the SLBS Computer Viruses Spread Model: Case Study in the Computer Laboratory, Diponegoro University, Indonesia

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A computer virus is a virus that attacks the computer that works by multiplying after a successful entry on a computer and aggravate the computer so that the computer becomes slow. The spread of computer viruses studied in the form of mathematical models is non linear with models SLBS (*Suspectible, Latent, Breaking-out)*. After that, look for basic reproduction number ratio (R_0). Then, we analize the stability of the disease-free and endemic equilibrium points. Further, numerical simulations carried out based on data from Computer Laboratory, Mathematics Departement, Diponeoro University. From the analysis is known that basic reproduction number ratio is obtained $R_0 > 1$, the stability of known free equilibrium is unstable, whereas the endemic equilibrium is locally asymptotically stable, which means there was an outbreak of computer virus. After that, we apply the optimal control strategies to minimize many breaking-out computers. From the simulation results found that many breaking-out computers comes down, it means that the spread of the computer viruses can be controled.

Keywords: Computer Virus, SLBS Model, Stability Analysis, Optimal Control.

1. INTRODUCTION

Computer technology is growing rapidly in this era of globalization. Computerized almost used in all areas. In addition, the computer is not a need for a group of people, but has become a device that can help solve someone's job. Therefore, someone needs to secure the data of interference or threats such as computer viruses.

The definition of a computer virus is a program or application on a computer that can damage a computer program or document contained corrupt data on a computer.[1] Computer viruses also have the ability to identify programs to be examined and infected, copy itself and infect, manipulation, and the ability to hide away.[2] As for certain parties are trying to make a virus with a view to exploit security system, such as passwords and bank account.

Model for computer viruses previously have been discussed by researchers. One of them by Yang et al.[3] which built a series of models of rational epidemic of computer viruses and developed a model spread computer viruses by proposing a new model SLBS. While that, Lijuan et al.[4] proposed a model SLBS about the spread of a computer virus with optimal control by the minimizing number of computers breaking-out.

Model spread of computer viruses has three classes, namely S (the number of vulnerable computers), L (the number of computers latent), and B (the number of computers breaking-out. Controlling is done by installing an efficient antivirus and firewall to computers breaking-out.

2. BRIEF REVIEW OF THE COMPUTER VIRUSES SPREAD MATHEMATICAL MODELING

Mathematical models have a role in analyzing a phenomenon of epidemic in the health field to determine the level of virus spread of infectious diseases. Contact between vulnerable computers infected with a virus infected computer virus, similar to the spread of epidemic contact between individuals are susceptible to the virus by the infected individual virus. So the endemic model was used to analyze the spread of computer viruses.[3] The mathematical model the spread of a computer virus is as follows:

$$
\begin{aligned}
\frac{dS}{dt} &= \mu_1 + \gamma_1 L + \gamma_2 B - (\beta_1 L + \beta_2 B)S - (\delta + \theta)S \\
\frac{dL}{dt} &= \mu_2 + (\beta_1 L + \beta_2 B)S + \theta S - (\alpha + \gamma_1 + \delta)L \qquad (1) \\
\frac{dB}{dt} &= \alpha L - (\gamma_2 + \delta)B
\end{aligned}
$$

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Controlling is done by installing antivirus and firewall programs are efficient for the computer breaking-out. Controlling the number of computers that evolved from the stages are susceptible to computer viruses to phase breaking-out an attempt to control the number of computer virus affected computer is to apply the controls u. The mathematical model of the SLBS computer viruses spread with control[4] as follows.

$$\frac{dS}{dt}=\mu_1+\gamma_1 L+\gamma_2 B-(\beta_1 L+\beta_2 B)S-(\delta+\theta)S+\omega uB$$
$$\frac{dL}{dt}=\mu_2+(\beta_1 L+\beta_2 B)S+\theta S-(\alpha+\gamma_1+\delta)L+(1-\omega)uB \quad (2)$$
$$\frac{dB}{dt}=\alpha L-(\gamma_2+\delta)B-uB$$

Further, consider model (1) that is a system of nonlinear differential equations with the initial condition has not been found computers infected with computer viruses. Equilibrium point[5] of the system can be searched by $dS/dt=0$, $dL/dt=0$, $dB/dt=0$. So that the system become

$$\mu_1+\gamma_1 L+\gamma_2 B-(\beta_1 L+\beta_2 B)S-(\delta+\theta)S=0 \quad (3)$$
$$\mu_2+(\beta_1 L+\beta_2 B)S+\theta S-(\alpha+\gamma_1+\delta)L=0 \quad (4)$$
$$\alpha L-(\gamma_2+\delta)B=0 \quad (5)$$

From Eqs. (3)–(5) can be obtained two equilibrium points of the model the spread of a computer virus, namely disease-free equilibrium point $(S^o, L^o, B^o)=(((\mu_1+\gamma_1)/(\delta+\theta)), 0, 0)$ and endemic equilibrium point

$$(S^*, L^*, B^*)=\left(\frac{XL^*-\mu_2}{(\beta_1 L^*+\beta_2(\alpha L^*/(\gamma_2+\delta))+\theta)},\ \frac{-n_1\pm\sqrt{n_1^2-4n_0 n_2}}{2n_0},\ \frac{\alpha L^*}{(\gamma_2+\delta)}\right)$$

with $X=(\alpha+\gamma_1+\delta)$.

3. RESULTS AND DISCUSSION

3.1. Stability Analysis of the Equilibrium Point

Before we analize the equilibrium point stability, consider basic reproduction numbers (R_0)[6] that represents the average number of newly infected individuals in the population are produced directly or indirectly from an infected individual during the period of the spread of the disease in vulnerable populations. Basic reproduction numbers (R_0) obtained by constructing a matrix that generates the number of new computers infected with Next Generation Matrix method (NGM)[7] and acquired the basic reproduction number is as follows

$$R_0=\frac{(\beta_1(\delta+\theta)+\alpha\beta_2)(\mu_1+\gamma_1)}{(\delta+\theta)(\alpha+\gamma_1+\delta)(\gamma_2+\delta)} \quad (6)$$

If $R_0<1$, means the infected computer virus will not spread and the number of infected computers on the wane. If $R_0>1$, means that infected computers can transmit computer viruses to other computers.

Furthermore, stability analysis[8] of mathematical models in the form of a system of nonlinear Eq. (1), can be determined by performing linearization. In this case, the linearized model is found by using first order Taylor series and we have a Jacobian matrix for disease-free equilibrium point as follows.

$$J(P_o)=\begin{bmatrix}-(\delta+\theta) & \left(\gamma_1-\beta_1\frac{\mu_1+\gamma_1}{\delta+\theta}\right) & \left(\gamma_2-\beta_2\frac{\mu_1+\gamma_1}{\delta+\theta}\right)\\ \theta & \left(\beta_1\frac{\mu_1+\gamma_1}{\delta+\theta}-\alpha-\gamma_1-\delta\right) & \beta_2\frac{\mu_1+\gamma_1}{\delta+\theta}\\ 0 & \alpha & -(\gamma_2+\delta)\end{bmatrix} \quad (7)$$

We obtained characteristic equation in the form of polynomial in λ,

$$h(\lambda)=q_3\lambda^3+q_2\lambda^2+q_1\lambda+q_0$$

with

$$q_3=1$$
$$q_2=a+h-e=\delta+\theta+\gamma_2+\delta-\beta_1\frac{\mu_1+\gamma_1}{(\delta+\theta)}-\alpha-\gamma_1-\delta$$
$$=\delta+\theta+\gamma_2-\beta_1\frac{\mu_1+\gamma_1}{(\delta+\theta)}-\alpha-\gamma_1$$
$$q_1=ah-ae-eh-fg-db$$
$$=(\delta+\theta)(\gamma_2+\delta)-(\delta+\theta)\left(\beta_1\frac{\mu_1+\gamma_1}{(\delta+\theta)}-\alpha-\gamma_1-\delta\right)$$
$$-\left(\beta_1\frac{\mu_1+\gamma_1}{(\delta+\theta)}-\alpha-\gamma_1-\delta\right)(\gamma_2+\delta)$$
$$\left(\beta_2\frac{\mu_1+\gamma_1}{(\delta+\theta)}\right)\alpha-\theta\left(\gamma_1-\beta_1\frac{\mu_1+\gamma_1}{(\delta+\theta)}\right)$$
$$q_0=dcg-aeh-afg-dbh=\theta\alpha\left(\gamma_2-\beta_2\frac{\mu_1+\gamma_1}{(\delta+\theta)}\right)$$
$$-(\delta+\theta)\left(\beta_1\frac{\mu_1+\gamma_1}{(\delta+\theta)}-\alpha-\gamma_1-\delta\right)(\gamma_2+\delta)$$
$$-(\delta+\theta)\left(\beta_2\frac{\mu_1+\gamma_1}{(\delta+\theta)}\right)\alpha-\theta\left(\gamma_1-\beta_1\frac{\mu_1+\gamma_1}{(\delta+\theta)}\right)(\gamma_2+\delta)$$

The stability of the disease-free equilibrium point is determined by using Routh-Hurwitz criteria.[9] The system is locally asymptotically stable, if it satisfies $q_0>0$, $q_1>0$, $q_2>0$, $q_3>0$, $(q_1q_2-q_0a_3)/a_2>0$ and $q_1q_2>q_0q_3$.

Furthermore, in the same way, we analyze the stability of the endemic-equilibrium point. Jacobian matrix for endemic equilibrium point is as follows:

$$J(P^*)=\begin{bmatrix}-\beta_1 w_2-\beta_2 w_3-\delta-\theta & \gamma_1-\beta_1 w_1 & \gamma_2-\beta_2 w_1\\ \beta_1 w_2+\beta_2 w_3+\theta & \beta_1 w_1-\alpha-\gamma_1-\delta & \beta_2 w_1\\ 0 & \alpha & -(\gamma_2+\delta)\end{bmatrix} \quad (8)$$

and we found a characteristic equation as follows.

$$h(\lambda)=p_3\lambda^3+p_2\lambda^2+p_1\lambda+p_0$$

with

$$p_3=1$$
$$p_2=a_1+h_1-e_1=\beta_1 w_2+\beta_2 w_3+\delta+\theta+\gamma_2+\delta-\beta_1 w_1-\alpha-\gamma_1-\delta$$

$$
\begin{aligned}
&= \beta_1 w_2 + \beta_2 w_3 + \delta + \theta + \gamma_2 - \beta_1 w_1 - \alpha - \gamma_1 \\
p_1 &= a_1 h_1 - a_1 e_1 - e_1 h_1 - f_1 g_1 - d_1 b_1 \\
&= (\beta_1 w_2 + \beta_2 w_3 + \delta + \theta)(\gamma_2 + \delta) \\
&\quad - (\beta_1 w_2 + \beta_2 w_3 + \delta + \theta)(\beta_1 S^* - \alpha - \gamma_1 - \delta) \\
&\quad - (\beta_1 S^* - \alpha - \gamma_1 - \delta)(\gamma_2 + \delta) - (\beta_2 w_1)\alpha \\
&\quad - (\beta_1 w_2 + \beta_2 w_3 + \theta)(\gamma_1 - \beta_1 w_1) \\
p_0 &= d_1 c_1 g_1 - a_1 e_1 h_1 - a_1 f_1 g_1 - d_1 b_1 h_1 \\
&= (\beta_1 w_2 + \beta_2 w_3 + \theta)\alpha(\gamma_2 - \beta_2 w_1) \\
&\quad - (\beta_1 w_2 + \beta_2 w_3 + \delta + \theta)(\beta_1 S^* - \alpha - \gamma_1 - \delta) \\
&\quad \times (\gamma_2 + \delta) - (\beta_1 w_2 + \beta_2 w_3 + \delta + \theta)(\beta_2 w_1)\alpha \\
&\quad - (\beta_1 w_2 + \beta_2 w_3 + \theta)(\gamma_1 - \beta_1 w_1)(\gamma_2 + \delta)
\end{aligned}
$$

The stability of the endemic-equilibrium point is determined by using Routh-Hurwitz criteria. The system is locally asymptotically stable, if it satisfies $p_0 > 0$, $p_1 > 0$, $p_2 > 0$, $p_3 > 0$, $(p_1 p_2 - p_0 p_3)/a_2 > 0$ and $p_1 p_2 > p_0 p_3$.

3.2. Optimal Control

Application of control with variable control u an attempt to prevent the spread of computer viruses by installing antivirus and firewall programs are efficient for the computer breaking-out. Therefore, the purpose of this optimal control problem is to minimize the number of computers breaking-out by minimizing the objective function is formulated as follows:

$$J(u) = \int_0^{tf} [B(t) + \frac{\varepsilon}{2} u^2(t)]\, dt \tag{9}$$

The first step is to find the equation of the Lagrangian and Hamilton optimum control[10] problem. Here is the Lagrangian equation of optimal control problem

$$L(B, u) = B(t) + \frac{\varepsilon}{2} u^2(t) \tag{10}$$

Then formed functional objective to minimize the Hamiltonian equation of Eqs. (2) and (10), that is:

$$
\begin{aligned}
&H(S, L, B, u, \lambda_1, \lambda_2, \lambda_3, t) \\
&\quad = L(B, t) + \lambda_1 \frac{dS}{dt} + \lambda_2 \frac{dL}{dt} + \lambda_3 \frac{dB}{dt}
\end{aligned} \tag{11}
$$

Thus obtained the Hamiltonian function is sought as follows:

$$
\begin{aligned}
H &= B + \frac{\varepsilon}{2} u^2 + \lambda_1(\mu_1 + \gamma_1 L + \gamma_2 B - (\beta_1 L + \beta_2 B)S \\
&\quad - (\delta + \theta)S + \omega u B) + \lambda_2(\mu_2 + (\beta_1 L + \beta_2 B)S \\
&\quad + \theta S - (\alpha + \gamma_1 + \delta)L + (1 - \omega)uB) \\
&\quad + \lambda_3(\alpha L - (\gamma_2 + \delta)B - uB)
\end{aligned} \tag{12}
$$

with λ_1, λ_2, λ_3 is the adjoint variables that can be identified by looking for equation costate and stationary conditions. Equation costate and stationary conditions can be searched using Pontryagin maximum principle as follows:

$$\lambda_1 = -\frac{\partial H}{\partial S} = \delta\lambda_1 + (\beta_1 L + \beta_2 B + \theta)(\lambda_1 - \lambda_2) \tag{13}$$

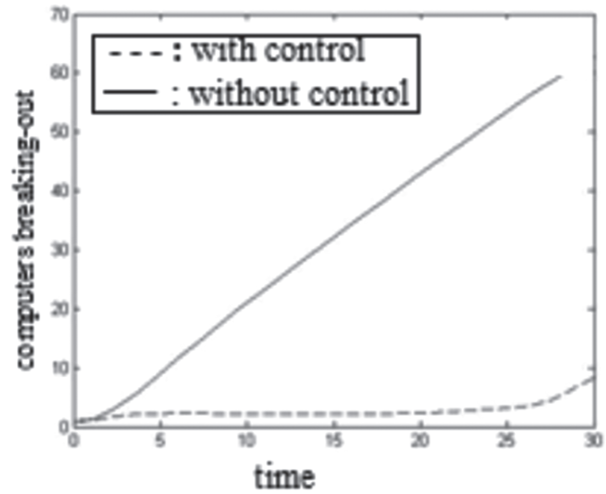

Fig. 1. The number of computers breaking-out without control and with control on the case I.

$$\lambda_2 = -\frac{\partial H}{\partial L} = \gamma_1(\lambda_2 - \lambda_1) + \beta_1 S(\lambda_1 - \lambda_2) - \alpha(\lambda_2 - \lambda_3) + \delta\lambda_2 \tag{14}$$

$$
\begin{aligned}
\lambda_3 = -\frac{\partial H}{\partial B} &= -1 + \gamma_2(\lambda_3 - \lambda_1) + (\beta_2 S - \omega u)(\lambda_1 - \lambda_2) \\
&\quad + u(\lambda_3 - \lambda_2) + \delta\lambda_3
\end{aligned} \tag{15}
$$

Hamilton optimizing conditions to form optimal control

$$\frac{\partial H}{\partial u} = \varepsilon u + \omega B\lambda_1 + (1 - \omega)B - B\lambda_3 \tag{16}$$

thus obtained

$$u^* = \frac{-\omega B\lambda_1 - (1 - \omega)B + B\lambda_3}{\varepsilon} \tag{17}$$

using properties obtained control room

$$u^* = \min\left(\max\left(\frac{-\omega B\lambda_1 - (1 - \omega)B + B\lambda_3}{\varepsilon}, \Delta\right)\right) \tag{18}$$

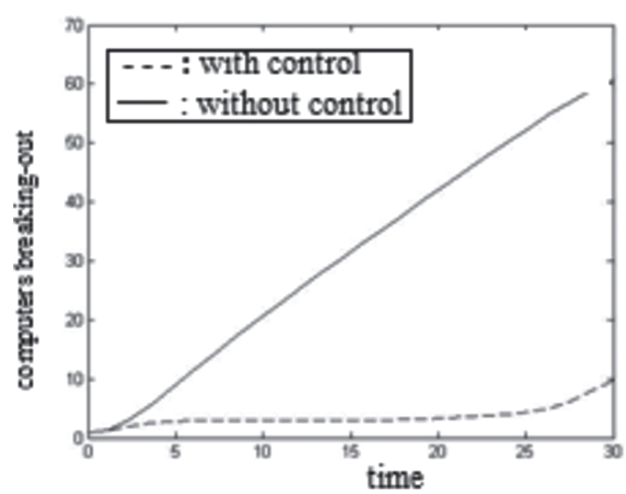

Fig. 2. The number of computers breaking-out without control and with control on the case II.

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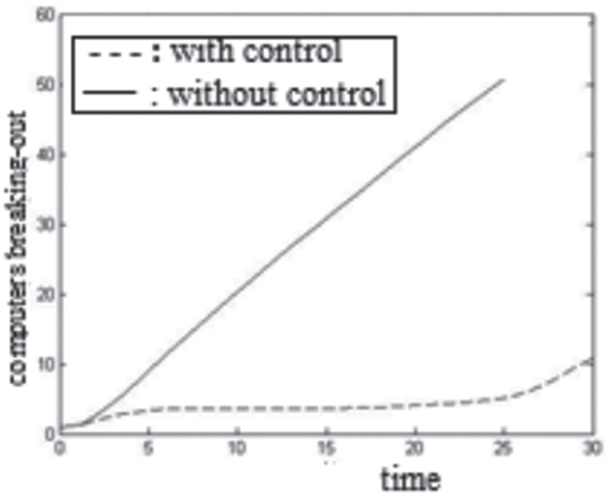

Fig. 3. The number of computers breaking-out without control and with control on the case III.

3.3. Simulation Results

Completion of the equation by using the method of Runge-Kutta Order 4. Without prejudice to the generality for numerical simulations, we used parameter values obtained by Chen Lijuan et al.,[4] i.e., $\mu_1 = 4$, $\mu_2 = 2$, $\beta_1 = 0.02$, $\beta_2 = 0.01$, $\gamma_1 = 0.1$, $\gamma_2 = 0.1$, $\delta = 0.1$, $\theta = 0.2$, $\alpha = 0.2$, $\omega = 0.8$ and based on data from the computer laboratory, Department of Mathematics, Diponegoro University, Indonesia, i.e., computers are susceptible = 20 computers, computers latent = 3 computers, and the number of computers breaking-out = 1 computer.

We obtained disease-free equilibrium point, $(S^o, L^o, B^o) = (14, 0, 0)$ and eigen values of the the Jacobian matrix are $-0{,}4$; 0,02 and $-0{,}48$, so that the disease-free equilibrium point is not stable. On the other hand, we have the endemic equilibrium point, $(S^*, L^*, B^*) = (9, 47; 18, 05; 18, 05)$ and eigen values of the Jacobian matrix for endemic equilibrium point re $-0{,}11$; $-0{,}39$; $-0{,}86$, so that the endemic equilibrium point is locally asymptotically stable.

Futhermore, we found basic reproduction number, $R_0 = 1, 12$ so that $R_0 > 1$, meaning that infected computers can transmit computer viruses to other computers. Therefore, given the control on the population spread of computer viruses.

To determine the effect on the population control the spread of computer viruses, performed simulations in the form of constant weight variation ε where ε to see the effect of the installation of efficient antiviruses and firewall to computers breaking-out. There are three cases in this regard, that is cases I ($\varepsilon = 10$), cases II ($\varepsilon = 20$), cases III ($\varepsilon = 30$). From the Figures 1–3, its can be seen that the control can minimize many breaking-out computers, so that the spread of computer viruses can be controlled.

4. CONCLUDING REMARKS

Based on the results of the discussion on the analysis of the stability that has been made known that the disease-free equilibrium point system is unstable and the endemic equilibrium point is locally asymptotically stable meaning that remains the spread of infection by viruses on the computer. Numerical simulations show the results of calculation of the basic reproduction ratio $R_0 = 1, 12$, so $R_0 > 1$ means a computer virus will spread and will become endemic.

In the simulation can be seen also the installation of efficient antiviruses and firewall to computers breaking-out can reduce the number of computers breaking-out, which means that the spread of computer viruses can be controlled. The smaller the weight vaues indicate that the reduction of the number of computers breaking-out will be greater.

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