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Optimal Strategy for Integrated Dynamic Inventory Control and Supplier Selection in Unknown Environment via Stochastic Dynamic Programming

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Abstract. In this paper, we propose a mathematical model in stochastic dynamic optimization form to determine the optimal strategy for an integrated single product inventory control problem and supplier selection problem where the demand and purchasing cost parameters are random. For each time period, by using the proposed model, we decide the optimal supplier and calculate the optimal product volume purchased from the optimal supplier so that the inventory level will be located at some point as close as possible to the reference point with minimal cost. We use stochastic dynamic programming to solve this problem and give several numerical experiments to evaluate the model. From the results, for each time period, the proposed model was generated the optimal supplier and the inventory level was tracked the reference point well.

Keywords: Inventory control problem, stochastic dynamic programming, supplier selection.

1. Introduction

Logistics and supply chain management involve optimal strategy determining to manage the procurement, movement and storage of materials, parts and finished product [1]. Inventory system and supplier selection are two of many components on logistics and supply chain management. Inventory level of a product has to be controlled for demand satisfying and the optimal supplier has to be selected if there are several alternatives. We can classify the environment of the inventory control problem into two class which are deterministic environment and stochastic environment.

Many researchers were developed the mathematical model to solve the inventory control problem in deterministic environment. For single product case, [2, 3, 4] were determined the optimal strategy for inventory controlling without supplier selection problem solving. In the other hand, [5, 6, 7] were solved dynamic supplier selection problem, but the inventory of the product was not controlled. Some researchers were integrated the inventory control problem and supplier selection problem where some assumptions were used. For example, [8] was solved the supplier selection problem integrated with inventory system but the inventory level was not controlled to follow the decision maker desiring, [9] was integrated production control with multi supplier and [10] was integrated the optimal control problem and supplier selection problem for single product under deterministic environment. Some researchers were also developed the mathematical model for multi-product inventory system, for example, [11] was solve the inventory control for it but the supplier selection problem was not solved and it was solved under deterministic environment.

In this paper, we will formulate a mathematical model in the stochastic dynamic optimization form to determine the optimal strategy for an integrated single product inventory control problem and supplier selection problem in unknown environment i.e. when the demand and purchasing cost parameters are random. We will use the stochastic dynamic programming to solve the proposed model to decide the optimal supplier from several alternatives and calculate the optimal product volume purchased from the optimal supplier so that the inventory level will be located at some point as close as possible to the reference point with minimal cost. The reference point will be decided by the decision maker. The Stochastic dynamic programming will be performed by LINGO optimization tool. We will give some numerical experiments to evaluate and simulate the model. From the results, for each time period, we will observe how the evolution of the inventory level compared to the reference point and who the optimal supplier is.

2. Another section of your paper

Let S denotes the number of the supplier. Let $X_{t,s}$ denotes the product volume purchased from supplier s at time period t. Let I_t denotes the inventory level of the product at time period t. The cost components in this problem are purchasing cost and holding cost. Let $U_{t,s}$ be the purchasing cost for the product per unit from supplier s at time period t and H_t be the holding cost per product's unit at time period t. Finally, let D_t denotes the random variable for the demand of the product.

To control the inventory level such that it will be as close as possible to a reference level r_t at time period t, we define the track reference objective $\left\|I_t - r_t\right\|_Q$ where Q is a quadratic semidefinite positive matrix and $\left\|x\right\|_Q$ means $x'Qx$ where x' is transpose of vector x. Let $Q = 1$, then we define the cost function as follows

$$\sum_{i=1}^{\Omega}\left[\mathrm{Pr}_i \cdot \left(\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{p=1}^{P}U_{t,s}X_{t,s} + \sum_{t=1}^{T}\sum_{p=1}^{P}H_tI_t + \sum_{t=1}^{T}\left(I_t - r_t\right)^2\right)\right] \tag{1}$$

where Pr_i be the probability of scenario i. To ensure that the demand of each product is satisfied at each time period, then we have the first constraint as follows

$$I_{t-1} + \sum_{s=1}^{S}X_{t,s} - I_t \geq D_t, \forall t \in T. \tag{2}$$

Suppose that supplier s has maximum capacity C_s to supply product for every time period t, then we have the second constraint as follows

$$X_{t,s} \leq C_s, \forall t \in T, \forall s \in S. \tag{3}$$

For every time period, if the inventory has maximum capacity M, then we have the third constraint as follows

$$I_t \leq M, \forall t \in T. \tag{4}$$

Integer constraint for the purchased product volume can be written as follows

$$X_{t,s} \in \{0,1,2,...\}, \forall t \in T, \forall s \in S. \tag{5}$$

Hence the mathematical model of this problem can be rewritten as follows

$$\min \sum_{i=1}^{\Omega} \left[\mathrm{Pr}_i \cdot \left(\sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{p=1}^{P} U_{t,s} X_{t,s} + \sum_{t=1}^{T} \sum_{p=1}^{P} H_t I_t + \sum_{t=1}^{T} \left(I_t - r_t \right)^2 \right) \right] \tag{6}$$

subject to:

$$I_{t-1} + \sum_{s=1}^{S} X_{t,s} - I_t \geq D_t, \forall t \in T, \tag{7}$$

$$X_{t,s} \leq C_s, \forall t \in T, \forall s \in S, \tag{8}$$

$$I_t \leq M, \forall t \in T, \tag{9}$$

$$X_{t,s} \in \{0,1,2,...\}, \forall t \in T, \forall s \in S. \tag{10}$$

3. Stochastic Dynamic Programming

Dynamic programming or multi-stage programming can be used to solve many optimization problems with several characteristics that are the problem can be divided into stages with the decision for each stage is required, each stage has several associate states, decision at a stage describe how the state at current stage is transformed into the state for the next stage, it has principle of optimality and there is recursion function for its stages [12]. Dynamic programming can be divided into two class which are deterministic, where all of information are known with certainty, and stochastic, where at least one parameter is uncertain/random. Stochastic dynamic programming can be solved by generating the scenarios of the problem and working forward/backward iteration.

To illustrate the stochastic dynamic programming, let t denote the stage usually time period of the problem, T denotes the number of the stage, x_t denotes the decision at time period t and Ω_t denotes the event space at time period t. The initial decision is taken in stage 0 and the recourse decisions is taken in stages 2, 3, ..., T. A scenario is used to express the possible outcomes, usually cost/profit, based on the realization of the probability distribution. A scenario is presenting one possible realization of the future and the enumeration of all possible combinations of outcomes. For discrete event space, all of scenarios, called scenario tree, can be illustrated by Figure (1). For continuous or discrete but infinite event space, the scenario tree can be generated by using a sampling method, Monte Carlo sampling for example, to approximate the possible outcomes using a finite scenario tree [13].
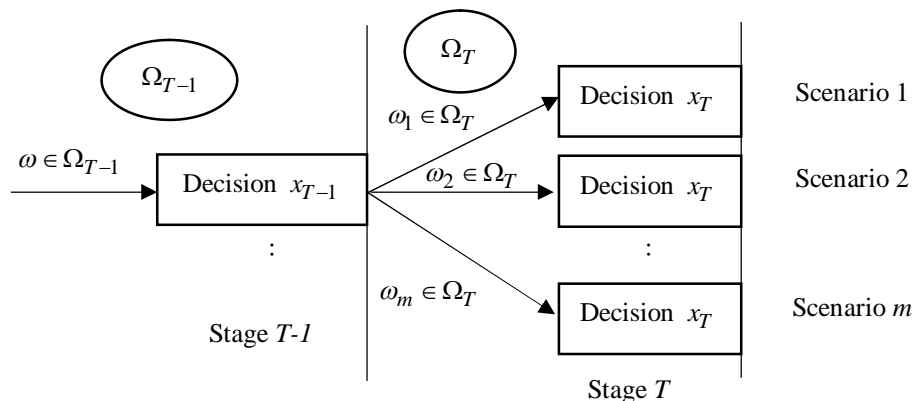

Figure 1. Scenario tree of a dynamic stochastic programming with discrete event space

Several optimization tools can be used to solve a dynamic programming by generating the scenario tree. In this paper, we use LINGO to solve the stochastic dynamic optimization (6).

4. Computational Simulation

4.1. The probability distribution is discrete

For the first numerical experiment, assume that the probability distribution of the random variables are discrete. Suppose that there are two suppliers namely s_1 and s_2. We will solve this problem for 3 periods. Suppose that the purchasing cost from supplier s_1 is \$12 per unit for each of time periods 1, 2 and 3. For time period-1, the purchasing cost from supplier s_2 is \$10 per unit, but for each of time periods 2 and 3, the purchasing cost is random with probability distribution given by Table (1).

Table 1. Probability distribution of purchase cost from s_2 for time period 2 and 3

Probability	Purchase cost (\$/unit/period)
0.5	10
0.5	12

The supplier capacity for each of suppliers s_1 and s_2 is 200 units per period. Let the initial inventory level is 0 item, the holding cost of the product is \$1/unit/period, the warehouse's maximum capacity is 200 units/period and the reference inventory level is 100 units for all time period. The demand for period 1 is known that is 100 units and the demand for each of periods 2 and 3 is random with the probability distribution given by Table (2).

Table 2. Demand's probability distribution for period 2 and 3

Probability	Demand (unit/period)
0.5	100
0.3	120
0.2	150

The decision maker desires that the inventory/stock level must be located at the reference level as close as possible with minimal cost. We solved the optimization problem (6) in Windows 8 AMD A6 2.7GHz and 2GB of Memory by using software LINGO 15.0 where the stochastic model class is multi-stage stochastic and the model class is MIQP (mixed integer quadratic programming). This stochastic dynamic program has 36 scenarios as shown by Table (3).

Table 3. Scenarios of the problem

Scenario	Time period (t)	Purchasing cost for s_2 (\$)	Demand (unit)	$X_{t,s}$		Inventory (unit)	Probability	Total Cost (\$)
				s_1	s_2			
1	1	12	100	0	199	99	0.0625	4270
	2	10	100	0	100	99		
	3	10	100	0	96	95		
2	1	12	100	0	199	99	0.0625	4490
	2	10	100	0	100	99		
	3	12	100	100	0	99		
3	1	12	100	0	199	99	0.0375	4470
	2	10	100	0	100	99		
	3	10	120	0	115	94		
4	1	12	100	0	199	99	0.0375	4700
	2	10	100	0	100	99		
	3	12	120	115	0	94		
⋮								
36	1	12	100	0	199	99	0.01	5862
	2	12	150	149	0	98		
	3	12	150	146	0	94		

The initial solution i.e. optimal strategy for time period 1 is 0 unit purchased from s_1 and 199 units purchased from s_2. For time period 2, the optimal strategy can be decided after the event of the random variables in time period 2 is revealed. Finally, the optimal strategy for time period 3 can be decided after the event of the random variables in time period 3 is revealed. From the result, the total expected cost is \$4819 obtained by the probability and the total cost for all scenarios, that is

$$0.0625 \cdot 4270 + 0.0625 \cdot 4490 + 0.0375 \cdot 4470 + 0.0375 \cdot 4700 + \cdots + 0.0100 \cdot 5862 = 4819.$$

To illustrate how the optimal strategy for each time period is decided, let the demand for period 2 is 100 units and purchasing cost for s_2 for time period 2 is \$10/unit, then we have to purchase 0 unit from s_1 and 100 units from s_2 and inventory level for period 2 is 99 units. Let the demand for period 3 is 120 units and purchasing cost for s_2 is \$12/unit, then we have to purchase 115 units from s_1 and purchase 0 unit from s_2. This solution is illustrated by Table (4).

Table 4. Illustration of the solution

Time period (t)	Purchasing cost for s_2 ($)	Demand (unit)	$X_{t,s}$		Inventory	Reference Inventory
			s_1	s_2		
1	12	100	0	199	99	100
2	10	100	0	100	99	100
3	12	120	115	0	94	100

From the comparison between inventory/stock level and the reference inventory level, it can be seen that the inventor/stock level of the product is followed the desired level well.

4.2. Demand's Probability Distribution is Continue

For the second example, suppose that the purchasing cost from s_2 for each of time periods 2 and 3 is normally distributed with mean 10 and standard deviation 4, the demand for each of periods 2 and 3 is normally distributed with mean 100 and standard deviation 20 and the remaining parameters are the same the first example. We solve this problem using LINGO where the scenarios are generated with Monte Carlo sampling where the sample size is 4. From the result, the total expected cost is \$4445.

For sensitivity analysis, suppose that the demand is normally distributed with mean 100 and standard deviation $\sigma > 0$. To observe the impact of the demand's standard deviation to the total expected cost, we evaluate the model with $\sigma = 5, 10, 15, 20, 25,$ and 30. The total expected cost is given by Figure (2).
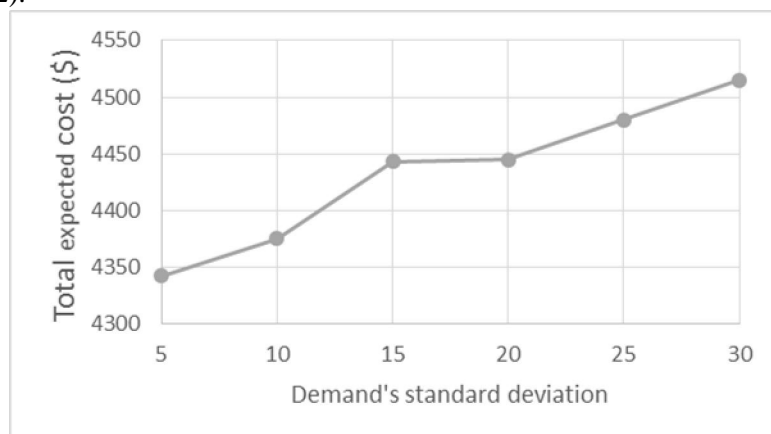

Figure 2. Impact of demand's standard deviation

From Figure (2), it can be seen that if the demand's standard deviation increases then the total expected cost is also increases. This is caused by the increasing of the demand uncertainty or the demand value has a wide range. If the demand's standard deviation more than 30, we predict that the total expected cost will be increased.

5. Concluding Remarks and Future Works

In this paper, the optimal strategy to control a single product dynamic inventory system integrated with supplier selection problem in unknown environment was considered. The mathematical model was written in the stochastic dynamic optimization and the optimal strategy was determined by using stochastic dynamic programming. Numerical experiments were considered with two cases which are the probability distribution of the random variables are discrete and continue in the normal probability distribution. From the results, it can be conclude that the supplier selection problem was solved and the inventory/stock level followed the desired level very well.

In the next works, we will develop the mathematical model for multi-product case along with order cost, penalty cost for defect or late delivery, service level, etc. Afterwards the random variable can be extended so that other parameters are random. In other works, we will develop the mathematical model so that it can also solve carrier selection problem.

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