Stability Analysis of Removal Nitrogen Model in Waste Stabilization Ponds

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This paper presents stability analysis of removal nitrogen model in waste stabilization ponds. The purpose of this paper is analysis the stability of removal nitrogen model in waste stabilization ponds. The theory of stability can be used for analyse stability of this model. The initial step in this method is find the equilibrium of this model. The equilibrium will be analysed. Furthermore, the process will be completed with the help of MAPLE program. Then, this model is analyzed stability. The results show that a decrease in the concentration of nitrogen every time. The analysis showed that this model is stable.

Keywords: Removal Nitrogen, Model, Stability Analyse, Waste Stabilization Ponds.

1. INTRODUCTION

Waste stabilization ponds is the ultimate solution to improve wastewater quality and widely used in developing countries. In wastewater treatment processes typically rely on the oxidation of organic material carried by bacteria using dissolved oxygen resulting from the photosynthesis performed by algae and surface re-aeration.

There are many chemical compounds contained in waste stabilization ponds, for example nitrogen, nitrate and ammonium. Increasing of the concentration of nitrogen in the waste stabilization ponds is undesirable because it can lead to ecological imbalance and affecting public health.[8] In wastewater treatment, the concentration levels of nitrogen are becoming increasingly important because nitrogen has many effects on the environment.[9] Similarly, the concentration levels of nitrate and ammonium. If the excessive concentration levels, it can cause a negative impact on the environment.

The increase in nitrogen concentration will have an impact on the amount of nitrate and ammonium, because the three chemical compounds that have a link between one and the other. In the nitrogen cycle process there are also processes that may affect the addition or reduction of the concentration of nitrate and ammonium. The existence of a significant increase in nitrogen concentration, will result in a significant change is also on the concentration of nitrate and ammonium.

Several studies on the removal nitrogen levels in the waste stabilization ponds has been done before. As Feirara et al.[4] discuss the dynamic models of nitrogen in waste stabilization ponds; Santos et al.[7] discuss about removal/reduction of nitrogen in an waste stabilization ponds in Portugal; Muttamara et al.[5] discuss the reduction of nitrogen in waste stabilization ponds baffled;

Senzia et al.[6] discuss the modeling of nitrogen transformation in primary facultative pond; Picot et al.[3] discusses the reduction of nitrogen in the stabilization ponds; Mayo et al.[1] discussed the nitrogen mass balance in waste stabilization ponds at the University of Dar Es Salam, Tanzania; Mayo et al.[2] investigated the removal of nitrogen in waste stabilization ponds; Mayo et al.[10] investigated the dynamic model of transformation and nitrogen removal in the pool of high level; Pirsaheb et al.[11] investigated the process variations of nitrogen and phosphorus compounds in waste stabilization ponds.

Removal nitrogen model in the waste stabilization ponds has been linked with high levels of nitrate and ammonium concentrations, then do research on the reduction of nitrogen levels in the waste stabilization ponds and associations with levels of nitrate and ammonium.

Urgency (virtue) of this study is to determine the reduced levels of nitrogen, nitrate and ammonium that can be used as an evaluation unit wastewater treatment process (WWTP).

2. EXPERIMENTAL DETAILS

Removal nitrogen model in Waste stabilization ponds are looked at are a model of organic nitrogen concentration changes, changes in the concentration of ammonia nitrogen, nitrate concentration change model. Model changes nitrogen in waste stabilization ponds made into one system by Mayo:[1]

$$\frac{d(N1)}{dt} = \frac{Q_i}{V}(N1) - \frac{Q_e}{V}(N1) - r_m - r_s + r_1 + r_2,$$

$$\frac{d(N2)}{dt} = \frac{Q_i}{V}(N2) - \frac{Q_e}{v}(N2) - r_1 + r_m - r_v - r_n, \qquad (1)$$

$$\frac{d(N3)}{dt} = \frac{Q_i}{V}(N3) - \frac{Q_e}{V}(N3) + r_n - r_d$$

then, $N1$: organic nitrogen, $N2$: ammonia, $N3$: nitrate.

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3. RESULTS AND DISCUSSION

Next, we will be determine the equilibrium point of the system (1). Condition of equilibrium obtained when its meet $dN1/dt = 0$, $dN2/dt = 0$, $dN3/dt = 0$, Let $(N1^*, N2^*, N3^*)$ stating the equilibrium point of the system (1), so that the system of equations at the point $(N1^*, N2^*, N3^*)$ can be written:

$$\frac{Q_i}{V}(N1^*) - \frac{Q_e}{V}(N1^*) - r_m - r_s + r_1 + r_2 = 0,$$
$$\frac{Q_i}{V}(N2^*) - \frac{Q_e}{V}(N2^*) - r_1 + r_m - r_v - r_n = 0,$$
$$\frac{Q_i}{V}(N3^*) - \frac{Q_e}{V}(N3^*) + r_n - r_d - r_2 = 0$$

Then we get

$$N1^* = \frac{(r_m + r_s - r_1)v}{Q_i - Q_e}, \quad N2^* = \frac{(r_1 - r_m + r_v + r_n)v}{Q_i - Q_e},$$
$$N3^* = \frac{(-r_n + r_d)v}{Q_i - Q_e}$$

From the calculation, the following one points of equilibrium, the equilibrium point

$$(N1^*, N2^*, N3^*)$$
$$= \left(\frac{(r_m + r_s - r_1)v}{Q_i - Q_e}, \frac{(r_1 - r_m + r_v + r_n)v}{Q_i - Q_e}, \frac{(-r_n + r_d)v}{Q_i - Q_e} \right)$$

The behavior of a system of linear equations (1) around the equilibrium point $(N1^*, N2^*, N3^*)$ can be determined by linear systems. Jacobian matrix of linearization obtained at the point of equilibrium $(N1^*, N2^*, N3^*)$, that is

$$J = \begin{bmatrix} \dfrac{Q_i - Q_e}{V} & 0 & 0 \\ 0 & \dfrac{Q_i - Q_e}{V} & 0 \\ 0 & 0 & \dfrac{Q_i - Q_e}{V} \end{bmatrix}$$
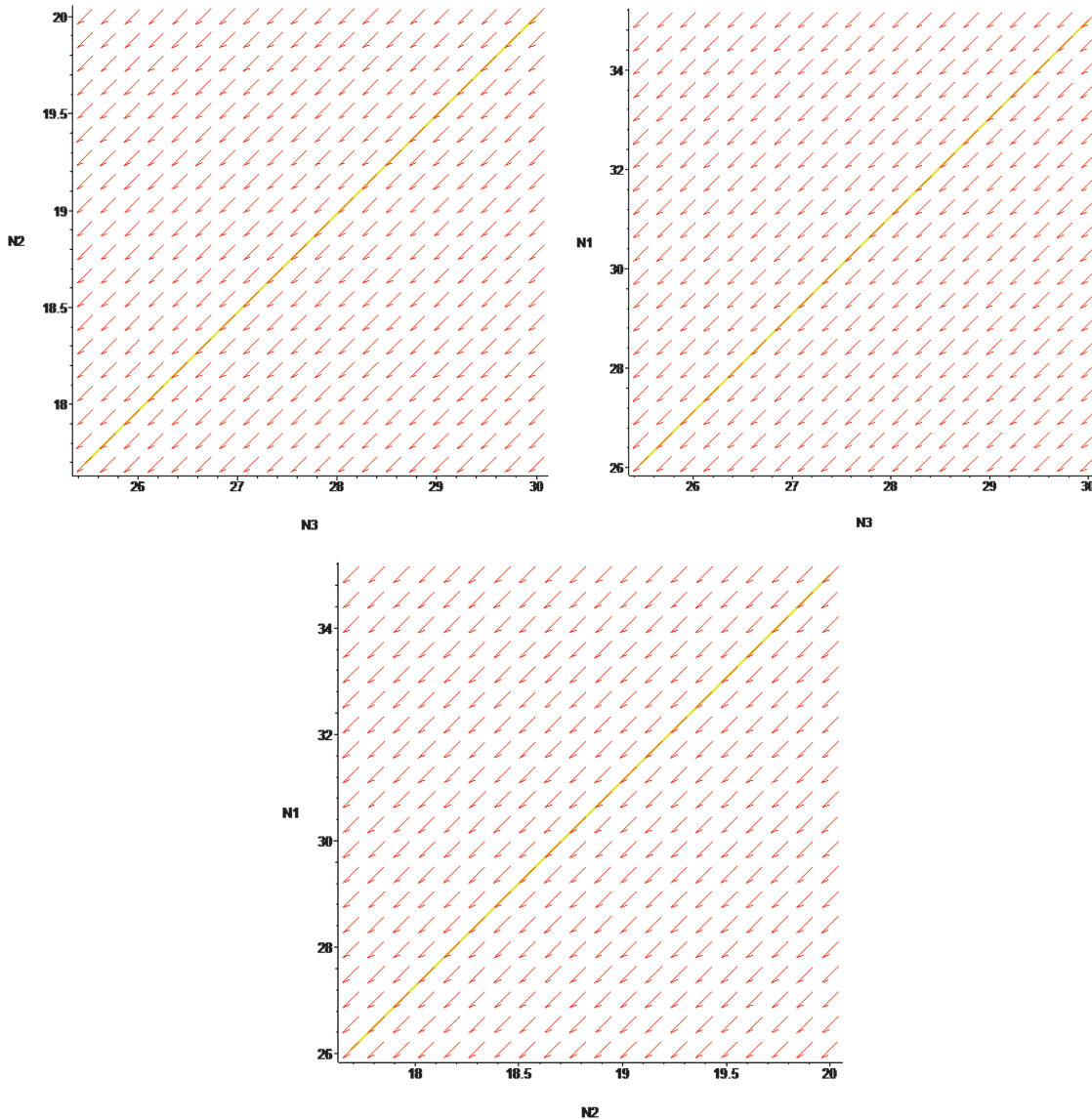

Fig. 1. Phase orbit around the equilibrium point.
Notes: In the figure shows that based on simulation, equilibrium point showed stable result. Stability condition of the equilibrium point in simulation is stable.

Then it will be analyzed the stability of the system by substituting the value of the equilibrium point. Substituting the point of equilibrium

$$\left(\frac{(r_m + r_s - r_1)v}{Q_i - Q_e}, \frac{(r_1 - r_m + r_v + r_n)v}{Q_i - Q_e}, \frac{(-r_n + r_d)v}{Q_i - Q_e} \right)$$

to the Jacobian matrix J to obtain

$$J = \begin{bmatrix} \dfrac{Q_i - Q_e}{V} & 0 & 0 \\ 0 & \dfrac{Q_i - Q_e}{V} & 0 \\ 0 & 0 & \dfrac{Q_i - Q_e}{V} \end{bmatrix}$$

By using det $|\lambda I - J| = 0$ and obtained characteristic equation

$$\left(\lambda - \frac{Q_i - Q_e}{V} \right)\left(\lambda - \frac{Q_i - Q_e}{V} \right)\left(\lambda - \frac{Q_i - Q_e}{V} \right) = 0$$

From this characteristics equation is obtained eigenvalues as follows,

$$\lambda_{1,2,3} = \frac{Q_i - Q_e}{V}$$

Thus, the stability of the equation depends on the value of root characteristics. Value $\lambda_{1,2,3} = (Q_i - Q_e)/V = 0$ for $Q_i = Q_e$. According to the theory of stability for the eigenvalues obtained is negative or $R_e(\lambda) \le 0$ then the equilibrium point

$$\left(\frac{(r_m + r_s - r_1)v}{Q_i - Q_e}, \frac{(r_1 - r_m + r_v + r_n)v}{Q_i - Q_e}, \frac{(-r_n + r_d)v}{Q_i - Q_e} \right)$$

on this system is stable.

In this section will be given a numerical simulation as the verification of the model of research that has been presented to use the data from Mayo.[1] $r_m = 1,149$, $r_s = 2,385$, $r_1 = 1,188$, $r_2 = 0,561$, $r_v = 0,043$, $r_n = 0,38$, $r_d = 0,527$. From the data obtained by the mathematical model of linear differential equations system

as follows Eq. (1). The simulations were being used to see field plot of the model.

4. CONCLUSION

In this paper, the model discussed is the growth model of dynamic system nitrogen, nitrate and ammonium. The form of the model is a dynamic system model linear. Further, we analyze local stability of the dynamical system based on the theory of stability if the eigenvalues obtained is negative or $R_e(\lambda) \le 0$ then the point of equilibrium in the system stable. However, if there exist the real part of the eigenvalues is positive or $R_e(\lambda) > 0$ then the equilibrium of system is unstable. From the simulation results is found that if the real part of the eigenvalues are negative and 0, so that the equilibrium point on the system are stable. It means that the longer the amount of nitrogen, ammonium and nitrate in waste stabilization ponds will be reduced.

References and Notes

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