



# Distribution Model 1-D of Concentration on Chemical Oxygen Demand in Waste Stabilization Ponds

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This paper presents solution of *Chemical Oxygen Demand* (COD) concentration distribution model on the surface waste stabilization ponds based on the advection–diffusion mechanism. This model is represented in second-order partial differential equations. The purpose of this paper is investigate the COD distribution design in waste stabilization ponds using field data obtained. Collection of field data was carried out in the Waste Water Treatment Plant (WWTP) Sewon, Bantul, D. I. Yogyakarta. Numerical method used for solution this model is finite difference method with Dufort Frankel scheme. The initial step in this method is process discretization by the finite difference schemes are used. The discrete equation will be substituted into the partial differential equations. Furthermore, the calculation will be completed with the help of MATLAB program. The results show that there was a mass transfer of pollutants each time followed by an increase and decrease in mass. This shows that there is a advection and diffusion process in waste stabilization ponds.

**Keywords:** Advection–Diffusion, Chemical Oxygen Demand, Dufort Frankel, Finite Difference, Waste Stabilization Ponds, WWTP Sewon.

## 1. INTRODUCTION

Waste stabilization ponds is used to improve quality of wastewater by relying on natural processes that treat wastewater by utilizing the presence of bacteria, algae, and zooplankton to reduce organic pollutants contained in the wastewater.<sup>1–3</sup>

Dependence on natural processes in the pond to degrade the waste water process makes the system vulnerable to environmental factors such as temperature, pH, sunlight, wind and other environmental factors. To understand and know the effect of waste water conditions on the process of degradation of organic material, namely an ability to degrade organic matter, then drafted a model distribution of the concentration of organic matter represented by the Chemical Oxygen Demand.

The various studies have been conducted about advection diffusion mechanism with analytically and numerically solution, such as Ref. [4] developed a numerical model to predict the flow and transport of fecal matter into the water surface. Then Ref. [5] is comparing some finite difference methods, namely FTCS, Upstream, Dufort Frankel and Crank Nicolson to solve equations one-dimensional advection dispersion. Reference [6]

complete the advection diffusion equation 2-D by using the finite difference method Dufort Frankel, and then<sup>7</sup> was combine Dufort Frankel with various other finite difference method. Reference [8] completed the advection—dispersion equation by using the three-dimensional finite difference method Forward Time Central Space (FTCS). Furthermore,<sup>9</sup> develop and compare several different numerical techniques to solve advection–diffusion equations in three dimensions.

Many research in waste stabilization ponds have also been carried out by the author, such as we are developed dynamic models on a facultative stabilization ponds with the phenomenon of the wastewater treatment process and then the model was used as a performance evaluation methods to WWTP.<sup>10</sup> Then we did the modeling environment in the domestic waste stabilization ponds facultative with the steady state.<sup>11</sup> Furthermore, we are developed the distribution model 1-D organic material that does not consider the depth of the ponds.

The wastewater treatment using COD (Chemical Oxygen Demand) as indicator to determine the quality of wastewater, so we conducted research on the distribution model 1-D of the concentration COD on waste stabilization ponds by advection–diffusion mechanism. In this case, the COD concentration on

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one side in the ponds was studied their changes through time. Changes in 1-D COD concentration through time can be represented in the form of partial differential equations. And because of the research was conducted in the stabilization ponds which are domain of simple field with a structured grid discretization process, so the numerical method used for the simulation of the model are finite difference method Dufort Frankel. Then, we use the data measurement of the concentration of the organic matter in the WWTP, Sewon, Bantul, to validation model.

## 2. EXPERIMENTAL DETAILS

### 2.1. Distribution Model of Pollutant Advection–Diffusion

Development of mathematical equations COD concentration on horizontally in columns pond can be described as advection–diffusion equation was formulated 1-D as follows:<sup>12</sup>

$$\frac{\partial C}{\partial t} = -\frac{\partial(uC)}{\partial x} + D_{mx} \frac{\partial^2 C}{\partial x^2} \quad (1)$$

### 2.2. Finite Difference Method Dufort Frankel

In general, basic finite difference method is to get the value of a variable as a function of space at step-time  $t + \Delta t$  by space distribution space at step-time to  $t$  known. The initial value is at zero step-time.

Validation of the model is done by using the measurement data on the concentrations of organic matter facultative stabilization ponds WWTP, Sewon, Bantul. Samples are measured on a facultative stabilization ponds II. Sampling points to be measured are; 3 points in the middle of the ponds with 3 points to the initial value, 2 points along the left and right side of ponds. Discretization a Dufort Frankel scheme shown in Figure 1.

Derivative approximation with respect to time with a center finite difference approach is as follows:<sup>13</sup>

$$\frac{\partial C}{\partial t}(i\Delta x, t\Delta t) = \frac{C_i^{t+1} - C_i^{t-1}}{\Delta t} - O(\Delta t) \quad (2)$$

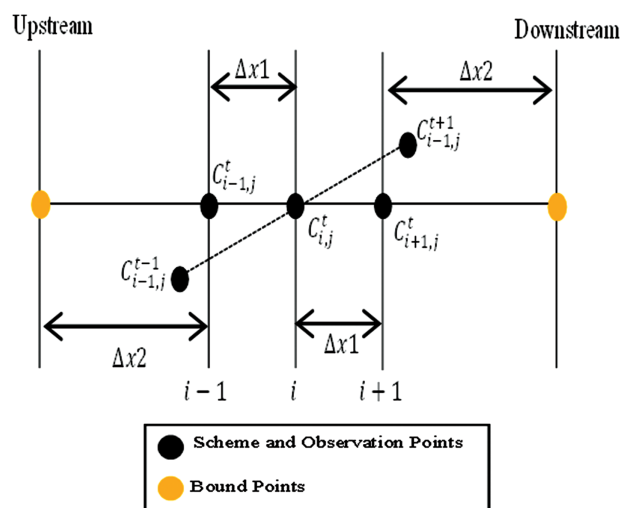


Fig. 1. Discretization ponds with Dufort Frankel Scheme.

Then, the derivatives space with a center finite difference approach are as follows:<sup>13</sup>

$$\frac{\partial C}{\partial x}(i\Delta x, t\Delta t) = \frac{C_{i+1}^t - C_{i-1}^t}{2\Delta x} - O(\Delta x^2) \quad (3)$$

$$\frac{\partial^2 C}{\partial x^2}(i\Delta x, t\Delta t) = \frac{C_{i+1}^t - C_i^{t+1} - C_i^{t-1} + C_{i-1}^t}{\Delta x^2} + O(\Delta x) \quad (4)$$

## 3. RESULTS AND DISCUSSION

Equation (1) is converted into a discrete form using the finite difference method Dufort Frankel scheme, by substituting the Eqs. (2)–(4) to the the Eq. (1), so that the results are as follows:

$$\frac{C_i^{t+1} - C_i^t}{\Delta t} = -u \frac{C_{i+1}^t - C_{i-1}^t}{2\Delta x} + D_{mx} \frac{C_{i+1}^t - C_i^{t+1} - C_i^{t-1} + C_{i-1}^t}{\Delta x^2} \quad (5)$$

By algebraic manipulations, the Eq. (5) can be written as follows:

$$C_i^{t+1} = C_{i-1}^t \frac{[A + B_x]}{[1 + B_x]} + C_i^{t-1} \frac{[1 - B_x]}{[1 + B_x]} + C_{i+1}^t \frac{[-A + B_x]}{[1 + B_x]} \quad (6)$$

Using the Eq. (6) will be calculated the value of  $C_i^{t+1}$  using a intial and boundary value as in Figure 1 to the desired iteration. Because of the calculation process is very long, it will be solved with the help of MATLAB. The initial value can be seen in Table I. And then, the boundary value can be seen in Table II. The boundary condition which used Dirichlet boundary condition. In this case, the boundary conditions assumed to be isolated, so that the boundary conditions of constant every time. To solve the equation, given the values of parameters of diffusion coefficient  $D_{mx} = 0.2 \text{ m}^2/\text{s}$ , the flow velocity  $u = 1.5 \text{ m/s}$  and the time flow  $t = 24 \text{ hours}$ . The given size grid  $\Delta x1 = 11$ ,  $\Delta x2 = 27.5$  and  $\Delta t = 1$ . Determination of the grid is adapted to the size of the pond, the length pond 77 meters.

The simulation results obtained are as follows.

Based on observations of Figure 2, it can be seen that there is a process of movement, the increase and decrease in mass. An increase in mass occurs because in this study only was assumed the ponds as a straight line. Then, we do the validation of models to compare COD concentration obtained from the numerical results with COD concentration of field research. Data were compared are at  $t = 10$ . Comparison of the research and numerical results can be seen in Table III.

We can see the differences between the research data with results of the calculation. The average of difference was 3.98%. The calculation of correlation coefficient was 0.973, meaning that 97% of the calculation data obtained same with the research data.

Table I. The initial value.

The grid	The initial value	
	$t = 0$	$t = 1$
$C_{2,3}$	124.69	125.81
$C_{3,3}$	158.60	127.50
$C_{4,3}$	117.46	118.65

Table II. The value of upper and lower boundary.

The right boundary	The value	The left boundary	The value
$C_{5,3}$	139.83	$C_{1,3}$	177.82

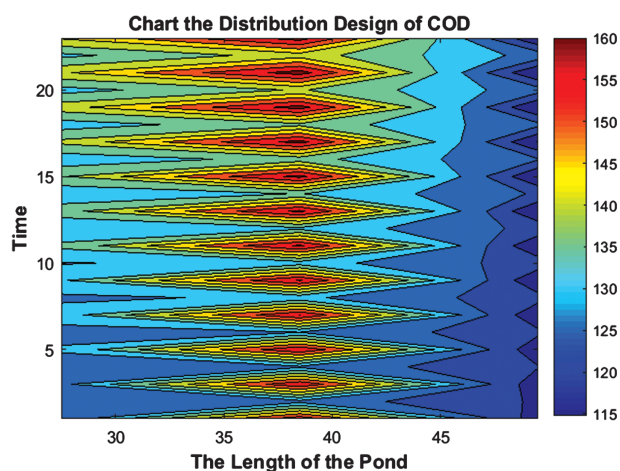


Fig. 2. Graph the distribution design of COD with  $t = 1$  until 24.

Table III. Research data and numerical results.

The grid	The initial value	
	$t = 0$	$t = 1$
$C_{2,3}$	124.69	125.81
$C_{3,3}$	158.60	127.50
$C_{4,3}$	117.46	118.65

#### 4. CONCLUSION

Based on the results and discussion, it is known that pollutants in a straight line or one side in waste stabilization ponds have advection and diffusion processes over time. From the results of

model validation is known that there is a difference between the calculation results with the research results. Nevertheless, the correlation between the calculation results with the research results tend very well because the correlation coefficient was 0.973.

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