# Implementation of lyapunov method to analyze the stability of pompano, cantang growth and nutrition dynamical systems

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## Implementation of lyapunov method to analyze the stability of pompano, cantang growth and nutrition dynamical systems

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**Abstract**. This study aims to determine the analysis stability of dynamical model of pompano, cantang growth, and nutrition on the Integrated Multi-Trophic Aquaculture (IMTA) systems by using the Lyapunov method. The analytical analysis was given to discuss the dynamic behavior of this model. Global stability analysis was performed based on the Lyapunov theory, i.e., construction the function V(x) that is a definite positive scalar function and the derivative of V(x) is defa inite negative. As a verification of the Lyapunov method, we conducted numerical simulations with data taken from IMTA systems in Sea Farming region, Kepulauan Seribu, Indonesia.

#### 1. Introduction

IMTA system is a cultivation practice with the maintenance of more than one species of organism or a polyculture system, but the cultivated species are those that have a mutualistic relation ecologically as a food chain in the area. Polyculture cultivation is now widely researched and studied because it can improve water quality. The integration of seaweed Eucheuma and Gracilaria sp sp into the activities of polyculture of tiger shrimp (Penaeus monodon) and milkfish (Chanos Chanos) in an integrated manner to get maximum yields than monoculture cultivation and more efficient use of land [1]

Cultivation is also widely developed in an attempt to fulfill the nutritional needs of humans [2]. The development of polyculture cultivation Integrated Multi-Trophic Aquaculture (IMTA) can increase economic benefits and minimize the negative impact of aquaculture on the environment [3], the impact of sediment and water quality [4]. During the process of growing fish need good nutrition. Feeding too much of an impact on the environment such as the analysis of water quality due to floating net cages [5, 6].

In the culture system IMTA types of marine life that are often cultivated as pompano fish farming and cantang star because the value of consumption is very high. The market demand for this fish is good enough both of the pompano and cantang besides the rapid growth and relatively high resale value [7].

There are many studies on the management cultivation of monoculture and polyculture system. However, a most mathematical model developed to predict the growth is a dynamical model in monoculture. On the other hand, the mathematical model for the polyculture system is still minimal with the stability analysis of this mathematical model only in the analysis of local stability around the

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equilibrium point [8]. So, we need a mathematical model to describe the growth of marine life more than one biota with a polyculture system and how to analyze the global stability of this model by using the Lyapunov method.

#### 2. Lyapunov Method to Analyze the Dynamical Model Stability

In the modeling process, the component of growth in IMTA system studied was composed of three variables, i.e. pompano, cantang, and nutrition. Three component interaction model, with a population of Pompano (B), cantang (M) and nutrition (P). Process of growth in the IMTA system in this case the system is modeled in equations (1). The skeleton of the think to obtain the model can be found at Sulpiani,et. al [8].

$$\frac{dP}{dt} = \theta - \left(\mu^{\frac{1}{3}}\rho\eta + \gamma^{\frac{1}{3}}\delta(1-\eta)\right)P$$

$$\frac{dB}{dt}$$

$$= \gamma^{\frac{1}{3}}(1-\eta)\delta PB^{\frac{2}{3}} - \varepsilon$$

$$+ \sigma M$$

$$\frac{dM}{dt} = \mu^{\frac{1}{3}}\eta\rho PM^{\frac{2}{3}} - \tau M - \sigma M$$

 $P \ge 0, B \ge 0, M \ge 0, \mu, \rho, \gamma, \delta, \varepsilon, \sigma, \tau > 0, 0 < \eta < 1,$ 

where  $\eta$  is nutrition consumed by cantang,  $1-\eta$  is nutrition consumed by pompano,  $\theta$  is nutrition into the cage,  $\sigma$  is the rest of cantang nutrition utilized pompano,  $\varepsilon$  is the nutrition necessary to maintain the viability of pompano,  $\tau$  is the nutrition necessary to maintain the viability of cantang,  $\gamma$  is constant stating the ratio of the pompano weight and length,  $\mu$  is constant stating the ratio of the cantang weight and length.

Next, the second Lyapunov method or often called by the Lyapunov direct method is used for globally stability analysis of the dynamical systems. This method commonly used to determine the stability of both linear and nonlinear systems, and does not need to find a solution from the system. The procedure of the Lyapunov method is given as follows.

- i. Let  $x_e = 0$  be an equilibrium point of  $\dot{x} = f(x, t), f: D \to R^n$
- ii. Let  $V: D \to R$  be a continuously differentiable function, such that
  - a. V(0) = 0
  - b. V(x) > 0, in  $D \{0\}$
  - c.  $\dot{V}(x) < 0$ , in  $D \{0\}$
- iii. Then  $x_e = 0$  is globally asymtotically stable

Meanwhile, theorem corresponding to the global stability of the system with Lyapunov function can be represented in Theorem 2.1.[9, 10].

#### Theorem 2.1

Let a system in the form of  $\dot{x} = f(x, t)$  with f(0, t) = 0 for all t. If there is a scalar function V(x) which has the first derivative and meets the following conditions

- 1. V(x) is positive definite,
- 2.  $\dot{V}(x)$  is negative define,

Then the equilibrium point is globally asymptotically stable. If it turns out that  $V(x) \to \infty$  for  $||x|| \to \infty$ , then the equilibrium point is globally asymptotically stable.

Further, V(x) is called by the Lyapunov function. Krasovskii method 10 one of the methods used to construct the Lyapunov function. From the dynamic system model (1) a Lyapunov function is constructed as follows,

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Equations (1) can be written 
$$f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} \dot{P} \\ \dot{B} \\ M \end{bmatrix}$$
, so that  $\begin{bmatrix} \dot{P} \\ \dot{B} \\ M \end{bmatrix} = \begin{bmatrix} \theta - \left(\mu^{\frac{1}{3}}\rho\eta + \gamma^{\frac{1}{3}}\delta(1-\eta)\right)P \\ \gamma^{\frac{1}{3}}(1-\eta)\delta PB^{\frac{2}{3}} - \varepsilon B + \sigma M \\ \mu^{\frac{1}{3}}\eta\rho PM^{\frac{2}{3}} - \tau M - \sigma M \end{bmatrix}$ .

Krasovskii defines Lyapunov's function as follows

$$V(\mathbf{x}) = f^{T}Rf,$$
where  $R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$ 

is a positive definite symmetric matrix. Then obtain

$$V(\mathbf{x}) = \begin{bmatrix} \theta - \left(\mu^{\frac{1}{3}}\rho\eta + \gamma^{\frac{1}{3}}\delta(1-\eta)\right)P \\ \gamma^{\frac{1}{3}}(1-\eta)\delta PB^{\frac{2}{3}} - \varepsilon B + \sigma M \\ \mu^{\frac{1}{3}}\eta\rho PM^{\frac{2}{3}} - \tau M - \sigma M \end{bmatrix}^{T} \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \cdot \begin{bmatrix} \theta - \left(\mu^{\frac{1}{3}}\rho\eta + \gamma^{\frac{1}{3}}\delta(1-\eta)\right)P \\ \frac{1}{\sqrt{3}}(1-\eta)\delta PB^{\frac{2}{3}} - \varepsilon B + \sigma M \\ \mu^{\frac{1}{3}}\eta\rho PM^{\frac{2}{3}} - \tau M - \sigma M \end{bmatrix}$$

$$V(\mathbf{x}) = \begin{bmatrix} \theta - \left(\mu^{\frac{1}{3}}\rho\eta + \gamma^{\frac{1}{3}}\delta(1-\eta)\right)P & \gamma^{\frac{1}{3}}(1-\eta)\delta PB^{\frac{2}{3}} - \varepsilon B + \sigma M & \mu^{\frac{1}{3}}\eta\rho PM^{\frac{2}{3}} - \tau M - \sigma M \end{bmatrix}$$

$$\cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \cdot \begin{bmatrix} \theta - \left(\mu^{\frac{1}{3}}\rho\eta + \gamma^{\frac{1}{3}}\delta(1-\eta)\right)P \\ \gamma^{\frac{1}{3}}(1-\eta)\delta PB^{\frac{2}{3}} - \varepsilon B + \sigma M \\ \mu^{\frac{1}{3}}\eta\rho PM^{\frac{2}{3}} - \tau M - \sigma M \end{bmatrix}$$

$$(2)$$

Furthermore, calculate the derivative of the Lyapunov function as follows, 
$$\dot{V}(\mathbf{x}) = \dot{f}^T R f + f^T R \dot{f}$$
, with  $\dot{f} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial t} = J f$ , and  $Q = J^T R + R J$ , Let  $\dot{f} = J f$ ,

with the Jacobian matrix, 
$$J = \begin{bmatrix} \frac{\partial f_1}{\partial P} & \frac{\partial f_1}{\partial B} & \frac{\partial f_1}{\partial M} \\ \frac{\partial f_2}{\partial P} & \frac{\partial f_2}{\partial B} & \frac{\partial f_2}{\partial M} \\ \frac{\partial f_3}{\partial P} & \frac{\partial f_3}{\partial B} & \frac{\partial f_3}{\partial M} \end{bmatrix}$$

$$J = \begin{bmatrix} -\left(\mu^{\frac{1}{3}}\rho\eta + \gamma^{\frac{1}{3}}\delta(1-\eta)\right) & 0 & 0 \\ \gamma^{\frac{1}{3}}(1-\eta)\delta B^{\frac{2}{3}} & \frac{2}{3}\gamma^{\frac{1}{3}}(1-\eta)P\delta B^{\frac{-1}{3}} - \varepsilon & \sigma \\ \mu^{\frac{1}{3}}\eta\rho M^{\frac{2}{3}} & 0 & \frac{2}{3}\mu^{\frac{1}{3}}\eta P\rho M^{\frac{-1}{3}} - \tau - \sigma \end{bmatrix}$$
Then obtain,

then obtain,

$$\dot{V}(\boldsymbol{x}) = f^T Q f$$

$$\dot{V}(\boldsymbol{x}) = f^T (J^T R + R J) f$$

$$\dot{V}(\boldsymbol{x}) = \int_{T}^{T} (J^T R + R J) f$$

$$\dot{V}(\boldsymbol{x}) = \int_{T}^{T} (J^T R + R J) f$$

$$\dot{V}(\boldsymbol{x}) = \begin{bmatrix} \theta - \left(\mu^{\frac{1}{3}}\rho\eta + \gamma^{\frac{1}{3}}\delta(1-\eta)\right)\rho \end{bmatrix}_{T}^{T} \begin{bmatrix} -\left(\mu^{\frac{1}{3}}\rho\eta + \gamma^{\frac{1}{3}}\delta(1-\eta)\right) & \gamma^{\frac{1}{3}}(1-\eta)\delta B^{\frac{2}{3}} & \mu^{\frac{1}{3}}\eta\rho M^{\frac{2}{3}} \\ 0 & \frac{2}{3}\gamma^{\frac{1}{3}}(1-\eta)\rho\delta B^{\frac{-1}{3}} - \varepsilon & 0 \\ 0 & \sigma & \frac{2}{3}\mu^{\frac{1}{3}}\eta\rho \rho M^{\frac{-1}{3}} - \tau - \sigma \end{bmatrix} \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$+\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} -\left(\mu^{\frac{1}{3}}\rho\eta + \gamma^{\frac{1}{3}}\delta(1-\eta)\right) & 0 & 0 \\ \gamma^{\frac{1}{3}}(1-\eta)\delta B^{\frac{2}{3}} & \frac{2}{3}\gamma^{\frac{1}{3}}(1-\eta)P\delta B^{\frac{-1}{3}} - \varepsilon & \sigma \\ \mu^{\frac{1}{3}}\eta\rho M^{\frac{2}{3}} & 0 & \frac{2}{3}\mu^{\frac{1}{3}}\eta P\rho M^{\frac{-1}{3}} - \tau - \sigma \end{bmatrix} \begin{bmatrix} \theta - \left(\mu^{\frac{1}{3}}\rho\eta + \gamma^{\frac{1}{3}}\delta(1-\eta)\right)P \\ \gamma^{\frac{1}{3}}(1-\eta)\delta PB^{\frac{2}{3}} - \varepsilon B + \sigma M \\ \mu^{\frac{1}{3}}\eta\rho PM^{\frac{2}{3}} - \tau M - \sigma M \end{bmatrix}$$
 (3)

#### 3. Results and Discussions

In this section is given a numerical simulation from the model (1) and construct the Lyapunov function to analyze the stability of the model at the equilibrium point. Based on data from the IMTA system in Sea Farming region, Kepulauan Seribu, Indonesia [11], the mathematical model is a nonlinear differential equations system as follows,

$$\frac{dP}{dt} = 1 - 2.778 P$$

$$\frac{dB}{dt} = 1.283 P B^{\frac{2}{3}} - 0.13B + 0.10M$$

$$\frac{dM}{dt} = 1.495 P M^{\frac{2}{3}} - 0.21M$$

The equilibrium point is obtained when fulfilling meet  $\frac{dP}{dt} = 0, \frac{dB}{dt} = 0, \frac{dM}{dt} = 0$ . Let  $(P^*, B^*, M^*)$  stating the equilibrium point of the model (4) so that we have the model of equations at the point  $(P^*, B^*, M^*)$  as follows,

$$\frac{dP}{dt} = 1 - 2.778 \ P^* = 0$$

$$\frac{dB}{dt} = 1.283 \ P^* B^{*\frac{2}{3}} - 0.13 B^* + 0.10 M^* = 0$$

$$\frac{dM}{dt} = 1.495 \ P^* M^{*\frac{2}{3}} - 0.21 M^* = 0$$
We found that of the equilibrium point is  $(P^*)$ 

We found that of the equilibrium point is  $(P^*, B^*, M^*) = (0.35,0,0)$ .

Further to analyze the global stability of a dynamic system model (4) at this equilibrium point, we apply the Lyapunov method. The lyapunov function is constructed as follows, Let equations depending on the time

$$\dot{x} = f(x, t)$$
, with  $x = \begin{bmatrix} \dot{P} \\ \dot{B} \\ \dot{M} \end{bmatrix}$ 

thus obtained

$$f_1 = 1 - 2.778 P$$
  
 $f_2 = 1.283 P B^{\frac{2}{3}} - 0.13B + 0.10M$   
 $f_3 = 1.495 P M^{\frac{2}{3}} - 0.21M$ 

moreover, can be written  $f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} \dot{P} \\ \dot{B} \\ M \end{bmatrix}$ , then

$$\begin{bmatrix} \dot{P} \\ \dot{B} \\ M \end{bmatrix} = \begin{bmatrix} 1 - 2.778 \ P \\ 1.283 \ P \ B^{\frac{2}{3}} - 0.13B + 0.10M \\ 1.495 \ P \ M^{\frac{2}{3}} - 0.21M \end{bmatrix}$$

Krasovskii defines the following Lyapunov function:

$$V(\mathbf{x}) = f^T R f$$

$$V(\mathbf{x}) = \begin{bmatrix} 1 - 2.778 P \\ 1.283 P B^{\frac{2}{3}} - 0.13B + 0.10M \\ 1.495 P M^{\frac{2}{3}} - 0.21M \end{bmatrix}^{T} \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} 1 - 2.778 P \\ 1.283 P B^{\frac{2}{3}} - 0.13B + 0.10M \\ 1.495 P M^{\frac{2}{3}} - 0.21M \end{bmatrix}$$

moreover, the derivative of the Lyapunov function,

$$+\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} -2.778 & 0 & 0 \\ 1.283 \, B_3^{\frac{2}{3}} & 1.855P \, B_3^{-\frac{1}{3}} - 0.13 & 0.1 \\ 1.495 \, M_3^{\frac{2}{3}} & 0 & 0.997P \, M_3^{-\frac{1}{3}} - 0.21 \end{bmatrix} \begin{bmatrix} 1 - 2.778 \, P \\ 1.283 \, P \, B_3^{\frac{2}{3}} - 0.13B + 0.10M \\ 1.494 \, P \, M_3^{\frac{2}{3}} - 0.21M \end{bmatrix}$$

Note that, by algebra manipulation, we found that V(x) is a positive definite scalar function and  $\dot{V}(x)$  is the definite negative. In addition, based on Theorem 2.1. we have  $V(x) \to \infty$  when  $||x|| \to \infty$ , so that the system is globally asymptotically stable. In this case we obtained globally asymptotically stable system which means that the cultivation practice of IMTA systems can be assumed the growth of pompano and cantang fish to normally run well.

#### Conclusion

In this paper, the model discussed is a dynamical system model of the pompano and cantang growth with nutrition as a significant factor in the cultivation of IMTA systems. Glearly stability analysis of dynamical systems obtained by the Lyapunov method, i.e., we construct the Lyapunov function which is definite positive, and the derivative of the Lyapunov function is definite negative. From the numerical significant systems of the pompano, cantang growth and nutrition at the equilibrium point is globally asymptotically stable. This indicates that pompano and cantang fish with sufficient nutrition can grow well in the IMTA system in Sea Farming region, Kepulauan Seribu, Indonesia.

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