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Optimal strategy for supplier selection problem integrated with optimal control problem of single product inventory system with piecewise holding cost

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Abstract. In this paper, we formulate a hybrid mathematical model of supplier selection problem integrated with inventory control problem of a single product inventory system with piecewise holding cost. This model will be formulated in a piecewise affine (PWA) form that can be converted into mixed logical dynamic (MLD) form. By using this MLD model, we solve the supplier selection problem and control this inventory system so that the stock level tracks a desired level as the reference trajectory as closed as possible with minimal total cost. We use model predictive control for hybrid system to solve the problem. From the numerical experiment results, the optimal supplier was selected at each time period and the evolution of the stock level tracks the desired level well.

1. Introduction

Management on supply chain tries to minimize the operational cost and improve the profit. It focuses in relationships in order to maximize outcome for all parties in the chain [1]. Supply chain network consisting of all parties that are manufacturer, suppliers, carriers or transporters, warehouses, retailers and customers [2]. The optimal strategy in supply chain management is made to minimize the operational cost with some service level constraint and determine the optimal quantity, optimal locations and optimal time of a product to be produced and distributed [3]. Supply chain contains inventory system that has to be optimized so that it gives the minimal holding cost. Inventory problem can appear in the several forms like raw material inventory, work-in-process inventory and finished product inventory and each of them needs to be controlled in order to minimize the holding cost [3]. Most of researchers were developed a mathematical model to optimize supply chain management. The basic mathematical model in a linear state space equation for periodic review inventory system was developed in [4] and the optimal strategy was determined by using linear quadratic control method. Another approach was used a particle swarm optimization to optimize the basic inventory system considering discount [5].

Supply chain management is also contains supplier selection problem. Supplier selection problem is appeared if we have several supplier alternatives to supply the product and we have to decide how many the product that will be purchased from each supplier. Several approaches were used to find the optimal strategy for supplier selection problem solving. The most approach is mathematical model [6]. In control theory, there is a mathematical model called hybrid dynamical system where the state value is depends on the current location (or event) which is called as piecewise-affine (PWA) model [7]. The PWA model can be converted into mixed logical dynamical system (MLD) by writing the PWA model in hybrid systems description language (HYSDEL) and the equivalent MLD model can be generated by using mld function in hybrid toolbox for MATLAB [8, 9]. For an inventory system with piecewise holding cost where the holding cost is different for several stock level intervals, [10] was formulated a hybrid mathematical model in piecewise-affine (PWA) model and controlled the stock level to track some desired level by using model predictive control for hybrid system. Predictive control is an optimal control method that can be used to solve a trajectory tracking control problem. The basic MPC for non-hybrid system was developed in [11] and the MPC for hybrid system was developed in [12, 13]. This optimal control method solves the problem i.e. determine the optimal input by defining an objective function and optimizing the corresponding optimization by using some optimization method.

In this paper, we integrate the supplier selection problem with inventory control problem of a single product inventory system with piecewise holding cost by formulating a PWA model and determine the optimal strategy by using model predictive control for hybrid system. We formulate the integrated supplier selection problem and inventory control problem in a PWA model, convert it into MLD model and determine the optimal strategy by using predictive control for hybrid system.

2. Mathematical Model

Let $u_s(k)$ be the amount of arriving shipment of the product from supplier s that arrived at time/review period k and n_p be the lead time delay. Then, the dynamic of stock level of the product $y(k) \geq 0$ for any $k \geq 0$ can be stated as follows

$$\begin{aligned} y(k) =& u_1(0) + u_2(0) + \cdots + u_S(0) \\ &+ u_1(1) + u_2(1) + \cdots + u_S(1) \\ &+ \cdots + u_1(k-n_p-1) + u_2(k-n_p-1) \\ &+ \cdots + u_m(k-n_p-1) - d(0) \\ &- d(1) - \cdots - d(k-1) \\ &= \sum_{j=0}^{k-n_p-1} \sum_{s=1}^{S} u_s(j) - \sum_{j=0}^{k-1} d(j). \end{aligned} \tag{1}$$

where S denotes the number of the suppliers and $d(k)$ denotes the demand of the product at review period k. We develop the model for inventory system with piecewise holding cost (or holding cost considering discount) as follows. Let $q_j, j = 1, 2, ..., m$ denotes the holding cost where

$$\text{holding cost} = \begin{cases} q_1, \text{ if } \hat{y}_0 = 0 \leq y(k) \leq \hat{y}_1 \\ q_2, \text{ if } \hat{y}_1 < y(k) \leq \hat{y}_2 \\ \ldots \\ q_m, \text{ if } \hat{y}_{m-1} < y(k) \leq \hat{y}_m. \end{cases} \tag{2}$$

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Let $x_1(k) = y(k)$, $x_i(k) = q_j u(k - n_p + i)$ for $i = 2, 3, ..., n_p + 1$ and

$$\begin{cases} x_1(k+1) = x_1(k) + x_2(k) - q_j d(k) \\ x_2(k+1) = x_3(k) \\ x_3(k+1) = x_4(k) \\ \vdots \\ x_{(n-1)}(k+1) = x_n(k) \\ x_n(k+1) = q_j u(k). \end{cases}$$

where $n = n_p + 1$, $x(k) = [x_1(k), x_2(k), ..., x_n(k)]'$, then the dynamic of stock level of the product can be written as a PWA model as follows

$$x(k+1) = \begin{cases} Ax(k) + B_1 u(k) + D_1 d(k) \text{ if } \hat{y}_0 \leq y(k) \leq \hat{y}_1 \\ Ax(k) + B_2 u(k) + D_2 d(k) \text{ if } \hat{y}_1 < y(k) \leq \hat{y}_2 \\ \vdots \\ Ax(k) + B_m u(k) \\ +D_m d(k) \text{ if } \hat{y}_{m-1} < y(k) \leq \hat{y}_m, \end{cases} \tag{3}$$

$$y(k) = \begin{cases} \frac{1}{q_1} x(k), \text{ if } \hat{y}_0 < y(k) \leq \hat{y}_1 \\ \vdots \\ \frac{1}{q_j} x(k), \text{ if } \hat{y}_{j-1} < y(k) \leq \hat{y}_j \\ \vdots \\ \frac{1}{q_m} x(k), \text{ if } \hat{y}_{m-1} < y(k) \leq \hat{y}_m. \end{cases} \tag{4}$$

where $A = \begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$, $B_j = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ q_j \end{bmatrix}$

and $D = \begin{bmatrix} -q_j \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$.

This PWA model can be controlled by using predictive control for hybrid system by converting it into equivalent MLD model then determining the optimal input by using mixed-integer quadratic programming.

3. Numerical Experiment

Suppose that there are four suppliers namely s_1, s_2, s_3 and s_4 to supply a product where the purchasing cost per unit product for each review period 1, 2, 3, ..., 80 from these suppliers is given by Table 1. The maximum capacity of each supplier to supply the product is given by Table 2.

Table 1. Purchasing cost per unit ($)

Time period (k)	s_1	s_2	s_3	s_4
1, 2, …, 20	9	9.5	10	10.5
21, 22, 23, …, 40	11	9	8	8
41, 42, 43, …., 60	10	11	12	8
61, 62, 63, …., 80	9	12	11	10

Table 2. Supplier capacity (unit)

Time period (k)	Supplier capacity			
	s_1	s_2	s_3	s_4
1, 2, 3, …, 80	150	200	250	300

Suppose that the holding cost per unit is \$25 if the product volume is 0 to 100 units and it will be \$20 if the product volume is more than 100 units, hence $q_1 = \$25$ and $q_2 = \$20$, the maximum capacity of the storage is 300 units and the lead time delay $n_p = 2$. We solved this problem in Windows 8 with AMD A6 2.7 GHz of processor and 4 GB of Memory by using software MATLAB R2013a with hybrid system toolbox. By writing PWA model (3)-(4) in HYSDEL (available in Appendix 1) and using *mld* function in hybrid system toolbox, we convert the PWA model (3)-(4) into equivalent MLD model by adding auxiliary states δ and z in the form of

$$x(k+1) = Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k) \tag{5}$$

$$y(k) = Cx(k) + D_1u(k) + D_2\delta(k) + D_3z(k) \tag{6}$$

$$E_2\delta(k) + E_3z(k) \leq E_1u(k) + E_4x(k) + E_5 \tag{7}$$

where the values of the matrices $A, B_1, B_2, B_3, C, D_1, D_2, D_3, E_1, E_2, E_3, E_4$ and E_5 are available in the Appendix 2. Let H_p be the length of the horizon prediction for controller designing by using model predictive control for hybrid system given by [9, 12], the objective function for this optimal control problem is defined as the gain of the actual stock level from the desired stock level with weighting matrix Q_y and the input cost with weighting matrix Q_u and this optimal control problem is defined as follows

$$\min_{[u,\delta,z]_0^{H_p-1}} \sum_{k=0}^{H_p-1} \left[\|y(k) - y_d\|_{Q_y}^2 + \|u(k)\|_{Q_u}^2 \right] \tag{8}$$

subject to :

$$\begin{cases} x(k+1) = Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k) \\ y(k) = Cx(k) + D_1u(k) + D_2\delta(k) + D_3z(k) \\ -E_4x(k) - E_1u(k) + E_2\delta(k) + E_3z(k) \leq E_5 \\ u_{\min} \leq u(k) \leq u_{\max} \\ x_{\min} \leq x(k) \leq x_{\max} \\ y_{\min} \leq y(k) \leq y_{\max} \end{cases}$$

where $k = 0, 1, ..., H_p - 1$, notations $v_{\min}$ and $v_{\max}$ mean the lower bound and upper bound for v respectively and the notation $\|v\|_Q^2$ means v^TQv. The value of the desired stock level y_d can be found in the simulation result. The demand for this simulation is assumed to be constant that is $d(k) = 20$ for all $k > 0$. Assume that the initial product stock level is 70 units and the other parameters are given by Table 3 and Table 4.

Table 3. Parameter value

Parameter	k	Q_y	H_p	$u_{\min}$	$u_{\max}$	$y_{\min}$	$y_{\max}$
Value	any	1	10	$\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix}150\\200\\250\\300\end{bmatrix}$	0	300

Table 4. Weighting matrices for input

Time period (k)	Q_u
1,2,...,20	$diag(9,9.5,10,10.5)$
21,22,...,40	$diag(11,9,8,8)$
41,42,...,60	$diag(10,11,12,8)$
61,62,...,80	$diag(9,12,11,10)$

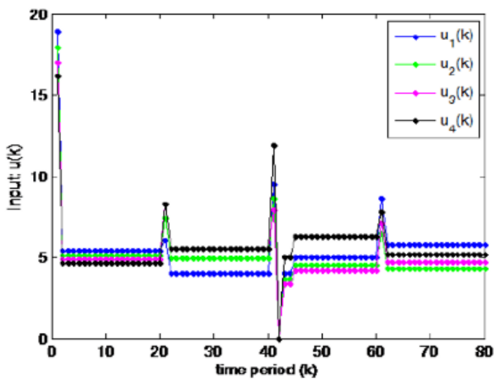

Figure 1. Optimal input $u(k)$

By solving (8) using mixed-integer quadratic programming, we have the simulation results shown by Fig. 1-2. The optimal input is shown by Fig. 1. The actual stock level $y(k)$ and the desired stock level $y_d(k)$ are shown by Fig. 2. Fig. 2 shows the optimal input u i.e. the amount of the arriving shipment of the product for each time or review period k. From this figure, it can be seen that all suppliers were selected to supply the product with different volume. This optimal input shown by Fig. 1 is applied to the system to generate the output i.e. the actual stock level $y(k)$ shown by Fig. 2. Fig. 2 shows the actual stock level and the desired stock level decided by decision maker. At time period 1 to 5, the product stock level was fluctuated due to the lead time delay is 2 periods and it has to satisfy the demand at time period 1 and 2. For time periods 5 to 80, the product stock level is following the desired level well although after the desired level is changed it needs 2 to 3 time periods to follow the desired level.

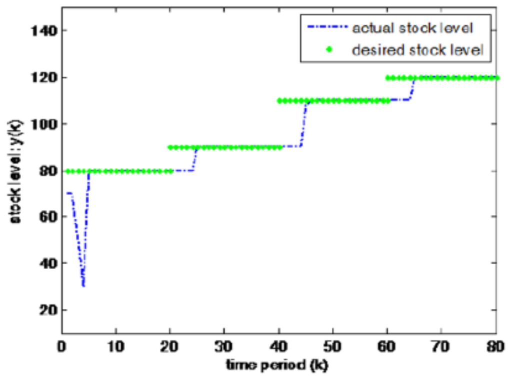

Figure 2. Output: actual stock level $y(k)$ and desired stock level y_d

4. Conclusion and Future Research

In this paper, the integrated supplier selection problem and tracking control problem of a single product inventory system with piecewise holding cost was considered. The problem was solved by formulating a piecewise affine model and determining the optimal amount of the product shipment from the optimal supplier by using model predictive control method for hybrid system. From the numerical experiment result, it can be conclude that the supplier selection problem was solved i.e. the optimal product volume from each supplier was determined and the stock level was controlled and followed the reference level well. In the future work, we will develop the model for multi-product inventory system and nonstationary demand. Furthermore, we will develop the model in deterministic and probabilistic environments.

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Appendix A. Code in HYSDEL for PWA model (3)-(4)

```
SYSTEM pwa_hybrid_inventory {
INTERFACE {
STATE { REAL x1 [0,4000];
        REAL x2 [0,4000];
        REAL x3 [0,4000];
        REAL y1 [0,200]; }
INPUT { REAL u1 [0,150];
        REAL u2 [0,200];
        REAL u3 [0,250];
        REAL u4 [0,300]; }
OUTPUT{ REAL y; }
PARAMETER { REAL y_hat_1, d, q1, q2; } }
IMPLEMENTATION {
AUX { REAL z1,z2,z3,z4;  BOOL d1; }
AD  { d1 = y1 >= y_hat_1; }
DA  { z1 = {IF d1 THEN  x1 + x2 - q2*d  ELSE  x1 + x2 - q1*d};
      z2 = {IF d1 THEN  x3  ELSE  x3 };
```

```
      z3 = {IF d1 THEN  q2*u1+q2*u2+q2*u3+q2*u4  ELSE   q1*u1+q1*u2+q1*u3+q1*u4 };
      z4 = {IF d1 THEN  x1/q2  ELSE  x1/q1 };  }
    CONTINUOUS { x1 = z1;   x2 = z2;  x3 = z3;  y1 = z4; }
    OUTPUT { y = y1;  } } }
```

Appendix 2. Matrices for system (5)-(7)

$A = zeros(4,4)$, $B_1 = zeros(4,4)$, $B_2 = zeros(4,1)$, $B_3 = eyes(4,4)$, $C = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$, $D_1 = zeros(1,4)$, $D_2 = 0$, $D_3 = zeros(1,4)$, $E_5^T = [100, 0, 8500, 7500, 500, -500, 4000, 4000, 0, 0, 18000, 22500, 0, 0, 200, 160, 0, 0]$, $E_2^T = [-100, 100, 8100, 7900, -7900, -8100, 4000, 4000, -4000, -4000, 18000, 22500, -22500, -18000, 200, 160, -160, -200]$,

$$E_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, E_4 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.5 & 0 & 0 & 0 \\ 0.05 & 0 & 0 & 0 \\ -0.04 & 0 & 0 & 0 \\ 0.04 & 0 & 0 & 0 \end{bmatrix}, E_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -20 & -20 & -20 & -20 \\ 20 & 20 & 20 & 20 \\ -25 & -25 & -25 & -25 \\ 25 & 25 & 25 & 25 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

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