Integrated Supplier Selection and Inventory Control of Probabilistic Single Product Inventory System with Product Discount and Its Optimal Strategy via Stochastic Dynamic Programming

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ABSTRACT

In this paper, we propose a stochastic dynamic optimization model to solve an integrated supplier selection and inventory control of single product inventory system with product discount. Bu using stochastic dynamic programming, we solve the problems which are determining the optimal supplier and bring the inventory level as close as possible to a reference point simultaneously with minimal cost. We give a numerical experiment to evaluate the model and to illustrate how the solution of the model is. From the result, the supplier selection was solved and the inventory level of the product is sufficiently closed to the reference point.

Keywords : Probabilistics Single Product, Inventory, Optimal Strategy, Stochastic Dynamic.

INTRODUCTION

Management on supply chain tries to reduce the cost so the manufacturers can improve their profit. There are many cost components that can be reduced on supply chain such as procurement cost, transportation cost and storage or inventory cost of a product [1]. To reduce the procurement cost and storage cost, manufacturers have to solve the supplier selection problem and control the inventory so these cost components give the minimal cost. Supplier selection problem is determining the optimal supplier from several alternatives and determining the optimal product volume purchased from the optimal supplier so that the demand is satisfied with minimal cost. Inventory control problem is determining the optimal product volume purchased from the supplier so that the inventory level is following some point decided by the decision maker with minimal cost. Many researchers were proposed a method to solve these problems. References [2, 3, 4] were solved the inventory control problem without solving the supplier selection problem. The solution from supplier selection problem was found in [5, 6, 7] without controlling the inventory level of the product. The integrated solution for supplier selection problem and inventory control problem was found in [8] but there is no product discount on the model. There are two problem classes on the supply chain management which are deterministic class where all of the parameters are known with certainty and probabilistic (or stochastic) class where at least one of the parameters is uncertain or random. Hence, we have to use a deterministic technique to solve the problem in deterministic environment and use a stochastic technique to solve the problem in probabilistic environment.

The most proposed method to manage the supply chain is mathematical model approach such as dynamic programming. In mathematical optimization theory, there are many optimization methods based on its class such as linear programming, quadratic programming, integer programming, dynamic programming, etc. where each of these methods can solve the problem in deterministic or probabilistic environment. The stochastic dynamic programming can be used to solve a dynamic optimization problem in probabilistic environment. Solving a dynamic optimization under uncertainty involves making optimal decision for a T-stage horizon before uncertain events are revealed while trying to maximize/minimize favorable/unfavorable outcomes that could be observed in the future [9, 10].

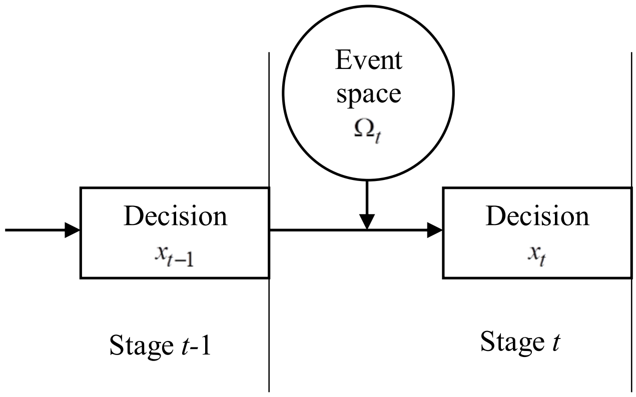

Figure 1. Relationship between the decision and realizations of random parameters

The stochastic dynamic programming can be illustrated as follows. Let t denote the stage or time period of the problem, x_t denotes the decision variable at time period t and Ω_t denotes the event space at any time period t. The relationship between the decision variables and realizations of random parameters can be illustrated by Figure (1). The initial decision is taken in stage 0 and the recourse decisions is taken in succeeding stages that can be interpreted as corrective decisions based on the past decisions taken and the realization of the actual values of the random parameters. A possible outcomes based on the realization of the probability distribution can be expressed as a scenario that presenting one possible realization of the future and the enumeration of all possible combinations of outcomes if all of parameters have discrete probability distribution can be represented as a scenario tree that can be illustrated by Figure (2).

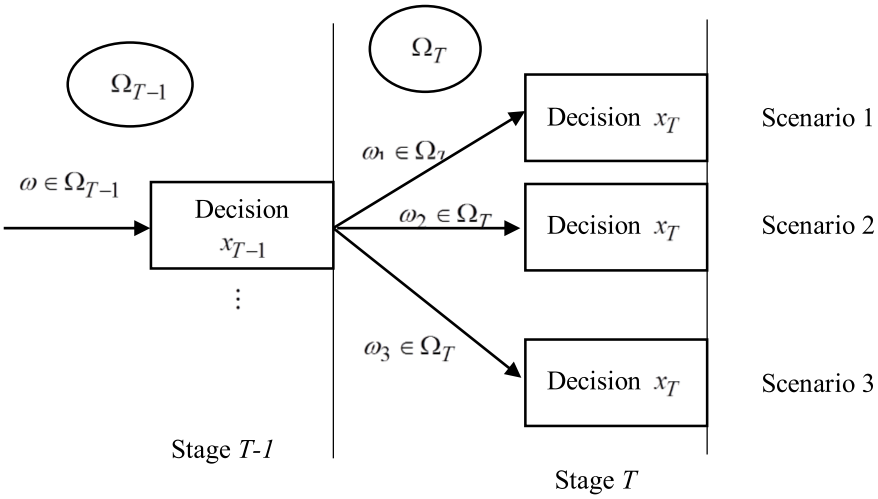

Figure 2. Scenario tree of a dynamic stochastic programming

For continuous or discrete but infinite event space, the scenario tree can be generated by using Monte Carlo sampling to approximate the problem using a finite scenario tree. Several optimization tools were developed that can be used to solve a stochastic dynamic optimization such as LINGO 15.0 [9].

In this paper, we propose a mathematical model in the form of stochastic dynamic optimization and use the stochastic dynamic programming to determine the optimal strategy for integrated supplier selection and inventory control problem of an inventory system with product discount in probabilistic environment. To evaluate and simulate the model, we give a numerical experiment with two suppliers where the purchasing cost and the demand are uncertain.

1. Mathematical Model

We formulate the mathematical model of the supplier selection and inventory system with product discount in probabilistic environment where the purchasing cost and the demand of the product are random with known probability distribution. Let P_i denotes the probability of scenario $i \in \Omega$ and Ω denotes the event space of the problem for any time period. Suppose that the number of the supplier is S and the number of the time period that the problem will be solved is T. Let $X_{t,s}$ denotes the product volume purchased from supplier s at time period t and $U_{t,s}$ denotes the purchasing cost per unit from supplier s at time period t. Let H_t denotes the holding cost per unit at time period t, D_t denotes the demand of the product at time period t and I_t denotes the inventory level of the product at time period t. The decision maker decides that the inventory level will be controlled so that it will be located at some point as close as possible to a reference point given by the decision maker. Let r_t denotes the reference point at time period t and $\left(I_t - r_t\right)^2$ be the reference tracking objectives. We minimize the total cost and the reference tracking objectives as follows

$$\min \begin{cases} \sum_{i=1}^{\Omega}\left(P_i \cdot \left[\sum_{t=1}^{T}\sum_{s=1}^{S} U_{t,s}^{(1)} X_{t,s} + \sum_{t=1}^{T} H_t I_t + \sum_{t=1}^{T}\left(I_t - r_t\right)^2\right]\right), & \text{if } d_s^{(0)} < X_{t,s} \le d_s^{(1)} \\ \sum_{i=1}^{\Omega}\left(P_i \cdot \left[\sum_{t=1}^{T}\sum_{s=1}^{S} U_{t,s}^{(2)} X_{t,s} + \sum_{t=1}^{T} H_t I_t + \sum_{t=1}^{T}\left(I_t - r_t\right)^2\right]\right), & \text{if } d_s^{(1)} < X_{t,s} \le d_s^{(1)} \\ \vdots & \\ \sum_{i=1}^{\Omega}\left(P_i \cdot \left[\sum_{t=1}^{T}\sum_{s=1}^{S} U_{t,s}^{(J)} X_{t,s} + \sum_{t=1}^{T} H_t I + \sum_{t=1}^{T}\left(I_t - r_t\right)^2\right]\right), & \text{if } d_s^{(J-1)} < X_{t,s} \le d_s^{(J)} \end{cases} \tag{1}$$

or it can be rewritten as follows

$$\min\left\{\sum_{i=1}^{\Omega}\left[P_i \cdot \left(\sum_{t=1}^{T}\sum_{s=1}^{S} U_{t,s}^{(j)} X_{t,s} + \sum_{t=1}^{T} H_t I_t + \sum_{t=1}^{T}\left(I_t - r_t\right)^2\right)\right], \quad \text{if } d_s^{(j-1)} < X_{t,s} \le d_s^{(j)}\right. \tag{2}$$

where the discount for purchasing cost is using the following scheme

$$U_{t,s} = \begin{cases} U_{t,s}^{(1)}, & \text{if } \quad d_s^{(0)} < X_{t,s} \le d_s^{(1)} \\ U_{t,s}^{(2)}, & \text{if } \quad d_s^{(1)} < X_{t,s} \le d_s^{(1)} \\ \vdots & \\ U_{t,s}^{(J)}, & \text{if } d_s^{(J-1)} < X_{t,s} \le d_s^{(J)} \end{cases}, \forall t \in T, \forall s \in S \tag{3}$$

or it can be rewritten as follows

$$U_{t,s} = U_{t,s}^{(j)}, \quad \text{if } d_s^{(j-1)} < X_{t,s} \le d_s^{(j)}, \forall t \in T, \forall s \in S \tag{4}$$

where $d_s^{(j)}, j = 0,1,2,...,J$ is the price level for this discount scheme.

The constraints of the model can be explained as follows. Constraint

$$I_{t-1} + \sum_{s=1}^{S} X_{t,s} - I_t \ge D_t, \forall t \in T \tag{5}$$

is used to ensure that the product in the storage and the purchased product will satisfy the demand. Constraint

$$X_{t,s} \le C_s, \forall t \in T, \forall s \in S \tag{6}$$

is used to ensure that the purchased product form supplier s is no more than the supplier capacity C_s. Let the maximum capacity of the storage is M , then constraint

$$I_t \le M, \forall t \in T \tag{7}$$

is used to ensure that the inventory level does not exceed the storage capacity. Finally, we have the last constraint which is integer constraint for the purchased product volume as follows

$$X_{t,s} \in \{0,1,2,...\}, \forall t \in T, \forall s \in S. \tag{8}$$

2. Numerical Experiment

Suppose that there are two suppliers which are s_1 and s_2 that will supply a product to a manufacturer. We solve the problem for 3 periods where all parameters in period 1 is known with certainty but the demand and purchasing cost from supplier s_2 for period 2 and 3 are random. The purchasing cost and supplier capacity are given by the Table 1 and Table 2. The purchasing cost $U_{t,2}^{(1)}$ and $U_{t,2}^{(2)}$ are random where their probability distribution is given by the Table 3 and Table 4.Tthe demand for period 1 is known which is is 90 units and the demand for each of period 2 and 3 is random with probability distribution given by Table 5. Let the initial inventory level is 0 item and the holding cost is \$1/unit/period. Finally, let the warehouse's maximum capacity is 200 units/period. The decision maker desires that the inventory/stock level must be located at some point as close as possible to the reference point which is 100 units for any time period with minimal cost.

TABLE 1. Purchasing cost & supplier capacity for time period 1

Supplier	Purchasing cost	Supplier capacity (unit/period)
s_1	$\begin{cases} \$11 & \text{if } X_{t,1} \le 100 \\ \$9 & \text{if } X_{t,1} > 100 \end{cases}$	150
s_2	$\begin{cases} \$13 & \text{if } X_{t,2} \le 150 \\ \$10 & \text{if } X_{t,2} > 150 \end{cases}$	200

TABLE 2. Purchasing cost & supplier capacity for time period 2 and 3

Supplier	Purchasing cost	Supplier capacity (unit/period)
s_1	$\begin{cases} \$11 & \text{if } X_{t,1} \le 100 \\ \$13 & \text{if } X_{t,1} > 100 \end{cases}$	150

s_2	$\begin{cases} \$U_{t,2}^{(1)} & \text{if } X_{t,2} \le 150 \\ \$U_{t,2}^{(2)} & \text{if } X_{t,2} > 150 \end{cases}$	200

TABLE 3. Probability distribution for $U_{t,2}^{(1)}$

Probability	Purchase cost $U_{t,2}^{(1)}$ (\$/unit/period)
0.6	12
0.4	10

TABLE 4. Probability distribution for $U_{t,2}^{(2)}$

Probability	Purchase cost $U_{t,2}^{(2)}$ (\$/unit/period)
0.3	10
0.7	9

TABLE 5. Demand's probability distribution for time period 2 and 3

Probability	Demand (unit/period)
0.3	80
0.7	100

We solve the optimization problem (1) in Windows 8 of OS, AMD A6 2.7 GHz of processor and 4 GB of memory by using LINGO 15.0 where the stochastic model class is multi-stage stochastic and the model class is PIQP (pure integer quadratic programming). By using stochastic dynamic programming method, we generate the scenario of the problem that was illustrated by Fig. 2. This problem has 64 scenarios as shown by Table 6. The value of the objective function (1) is called as the expected total cost which is the sum of the multiplication of the probability of the scenario and the total cost of the scenario. The minimal total expected cost of this problem is \$3716 which given by $0.0029 \cdot \$3489 + 0.0068 \cdot \$3644 + 0.0068 \cdot \$3465 + \cdots + 0.038 \cdot \$3824$.

TABLE 6. Scenarios of the problem

Scenario	Time period (t)	Purchasing cost for s_2 $\left(U_{t,2}^{(1)}, U_{t,2}^{(2)}\right)$ (\$)	Demand (unit)	$X_{t,s}$ s_1	$X_{t,s}$ s_2	Inventory (unit)	Probability	Total Cost (\$)
1	1	(13,10)	90	150	39	99	0.0029	3489
	2	(12,10)	80	81	0	100		
	3	(12,10)	80	80	0	100		
2	1	(13,10)	90	150	39	99	0.0068	3644
	2	(12,10)	80	81	0	100		
	3	(12,10)	100	95	0	95		
3	1	(13,10)	90	150	39	99	0.0068	3465
	2	(12,10)	80	81	0	100		

	3	(12,9)	80	76	0	96		
				⋮				
36	1	(13,10)	90	150	39	99	0.038	3824
	2	(10,9)	100	0	101	100		
	3	(10,9)	100	95	0	95		

From Table 6, it can be seen that the initial solution i.e. solution for time period 1 is 150 units purchased from s_1 and 39 units purchased from s_2 that will give 99 unit in the inventory. The optimal strategy for time period 2 and 3 can be illustrated by Table 8 after the random variables for time period 2 and 3 are revealed.

TABLE 8. Illustration of the solution

Time period (t)	**Purchasing cost for s_2 $\left(U_{t,2}^{(1)}, U_{t,2}^{(2)}\right)$ in \$**	**Demand (unit)**	**$X_{t,s}$**		**Inventory**	**Reference Inventory**
			s_1	s_2		
1	(13,10)	100	150	39	99	100
2	(12,10)	80	81	0	100	100
3	(10,9)	100	95	0	95	100

The optimal strategy for time period 2 can be decided after the random variables at time period 2 are revealed. Let the demand for period 2 is 80 units and purchasing cost for s_2 for time period 2 is (\$12, \$10), then we have to purchase 81 units from s_1 and 0 unit from s_2 and inventory level for period 2 is 100 units. The optimal strategy for time period 3 can be decided after the random variables at time period 3 are revealed. Let the demand for period 3 is 100 units and purchasing cost at time period 3 from supplier s_2 is (\$10, \$9), then we have to purchase 95 units from s_1 and purchase 0 unit from s_2 that will give 95 units in the inventory. From the comparison between inventory/stock level and the reference inventory level, it can be seen that the inventor/stock level of the product is followed the desired level well.

For the second experiment, let the probability distribution for $U_{2,2}^{(1)}$ is normal with mean 10 and variance 1 and the probability distribution for $U_{3,2}^{(2)}$ is normal with mean 10 and variance 2. The demand's probability distribution is also normal with mean 100 and variance 10. From the results, the expected total cost is \$4445.

CONCLUDING REMARKS

In this paper, a stochastic dynamic optimization model was considered to solve an integrated supplier selection problem and tracking control problem of single product inventory system with product discount in probabilistic environment. The optimal strategy was determined by using stochastic dynamic programming. Numerical experiments were considered with discrete and continuous probability distributions. From the results, it can be conclude that the supplier selection problem was solved and the inventory/stock level followed the desired level well.

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