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Linear Programming with Fuzzy Variable Method for Solving Wastewater Treatment Plant (WWTP) Problem

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Abstract. The waste load of organic matters in the stabilization ponds is degraded through biological treatment, that is to reduce or decrease the level of pollutants of organic matters by using and utilizing microorganisms. This problem will be solved using a mathematical model, that is linear programming with fuzzy variables on the Wastewater Treatment Plant (WWTP). The method in the case of Fuzzy Linear Programming (FLP) maximization is done by using simplex fuzzy method. The results of this method can be used as an evaluation method to determine the optimal waste load and the level of degradation of organic matter.

1. Introduction
The decreasing quality of the environment to date is still a problem that requires special attention. One of the causes of environmental degradation is the disposal of liquid waste, both domestic and industrial waste. The continuous disposal of liquid waste will cause environmental pollution, which of course will have a negative impact on people's lives. The problem of disposal of liquid waste will be a serious problem if the drainage is flowed without processing in advance. To minimize problems that occur, the wastewater needs to undergo a treatment process before being flowed into the water system. In general, the treatment process is carried out biologically, which uses the activity of micro-organisms that will decompose the pollutants of organic compounds contained in the waste. One biological treatment process is carried out in a waste stabilization pond (Waste Stabilization Ponds). According to Senzia et al. [1], Agunwamba et al. [2], Mara [3], Hamzeh and Ponce [4] the lagoon system is recommended for developing countries with tropical and subtropical climates. The stabilization pond consisting of maturation and facultative [5].

The quality of wastewater is improved in a stabilization pond so that the processed products meet the established quality standards and do not pollute the receiving water body. The quality standard of domestic wastewater is the size of the limit or level of pollutant elements and or the number of pollutant elements which are tolerated in their presence in domestic wastewater which will be discharged or released into surface water. This study aimed to determine the optimal waste load and degradation rate of organic matter. The mathematical

method used is using a linear fuzzy variable model, so that the results meet water quality standards.

2. Materials and Methods

2.1. Wastewater Treatment Plant (WWTP)

Waste treatment plants are used to prevent or reduce pollution to groundwater and to prevent germs caused by impurities that contaminate surface water. The waste stabilization pond that will be discussed here is the Sewon's WWTP, Bantul, Yogyakarta. The process of wastewater treatment at the Sewon's WWTP can be presented in the flow diagram of the operation of the Sewon's WWTP as in Figure 1.
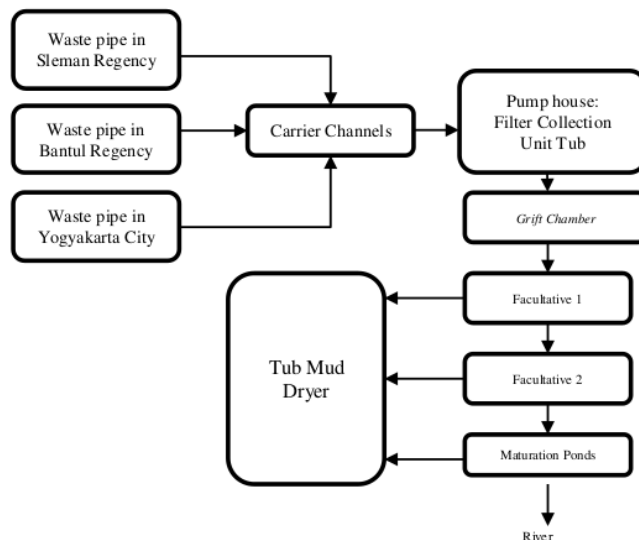

Figure 1. Diagram of the operation of the Sewon WWTP

The waste stabilization pond basically functions to improve the quality of wastewater so that the quality of its processed products meets the established quality standards and does not pollute the receiving water body. Water quality standards can be known by observing several parameters, one of which is Biochemical Oxygen Demand (BOD). BOD is the amount of oxygen needed by microorganisms to oxidize organic compounds in wastewater. Water with a high BOD value indicates a high number of pollutants, especially pollutants caused by organic matter. The BOD parameter is the most widely used parameter in wastewater testing.

2.2. Fuzzy Linear Programming

Fuzzy Linear Programming (FLP) formulation was first proposed by Zimmerman [8]. The fuzzy linear program is a linear program which is expressed by objective functions and constraint functions which have fuzzy parameters and fuzzy inequalities. The fuzzy linear program requires steps in finding the solution where in the fuzzy linear program approach to an ordinary linear program with a single objective function, maximum criteria are used. The general form of the fuzzy linear program model is as follows:

Objective function

$$\text{Max } \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j x_j$$

$$\text{Subject to } \sum_{j=1}^{n} \tilde{a}_{ij} x_j \lesssim \tilde{b}_i \qquad (i = 1,2, \dots, m)$$

$$x_j \geq 0 \qquad 1 \leq j \leq m \qquad\qquad (1)$$

with

x_j : decision variable
$\tilde{a}_{ij}$: technical coefficients in the form of fuzzy numbers
$\tilde{b}_i$: right segment coefficient in the form of fuzzy numbers
$\tilde{c}_j$: cost coefficient in the form of fuzzy numbers

There are many forms of problems. One of the fuzzy linear programs is a fuzzy linear program with technical coefficients in the form of fuzzy numbers. Problems Fuzzy linear programs with right-hand coefficients in the form of fuzzy numbers are presented by determining the limits and objective functions to be achieved from the decision variables in the form of linear inequalities, is:

Objective function

$\text{Max } \tilde{Z} = \sum_{j=1}^{n} c_j \tilde{x}_j$

Subject to $\sum_{j=1}^{n} a_{ij} \tilde{x}_j \lesssim \tilde{b}_i$ $(i = 1,2,\dots,m)$

                $x_j \geq 0$ $1 \leq j \leq m$ (2)

with

$\tilde{x}_j$: decision variables in the form of fuzzy numbers
a_{ij} : crisp coefficient
$\tilde{b}_i$: right segment coefficient in the form of fuzzy numbers
c_j : crisp coefficient

2.3. *Fuzzy Variable Linear programming (FVLP)*

In general, the form of Fuzzy Variable Linear Programming (FVLP) is formulated by determining the value of p variable x_j which meets the following m inequality or linear equation [10]:

Objective function

$\text{Maks/min } \tilde{Z} = c_1 \tilde{x}_1 + c_2 \tilde{x}_2 + \cdots + c_p \tilde{x}_p$

Subject to

$$a_{i_1}\tilde{x}_1 + a_{i_2}\tilde{x}_2 + \cdots + a_{i_p}\tilde{x}_p (\lesssim, \simeq, \gtrsim)\tilde{b}_i \qquad (i = 1,2,\dots,m)$$

$\tilde{x}_j \gtrsim \tilde{0}$ $j = 1,2,\dots,p$ (3)

The steps to complete a linear program model with fuzzy variables can be seen in Figure 2.

      Fuzzy Linear Programming mathematical model on wastewater treatment systems at Sewon WWTP, Bantul, Yogyakarta. Used as a method to model the optimal waste load of facultative ponds. Mathematical models in the form of inequalities and Linear Program equations with fuzzy variables. Optimization in terms of waste load processed in facultative ponds by taking into account the parameters of Biochemical Oxygen Demand (BOD) so that it meets water quality standards.

      The first step is to formulate a Mathematical model of objective and constraints. Then the model is formed into a mathematical equation and inequality by combining the concept of fuzzy logic in the Linear Program problem known as Fuzzy Linear Programming.
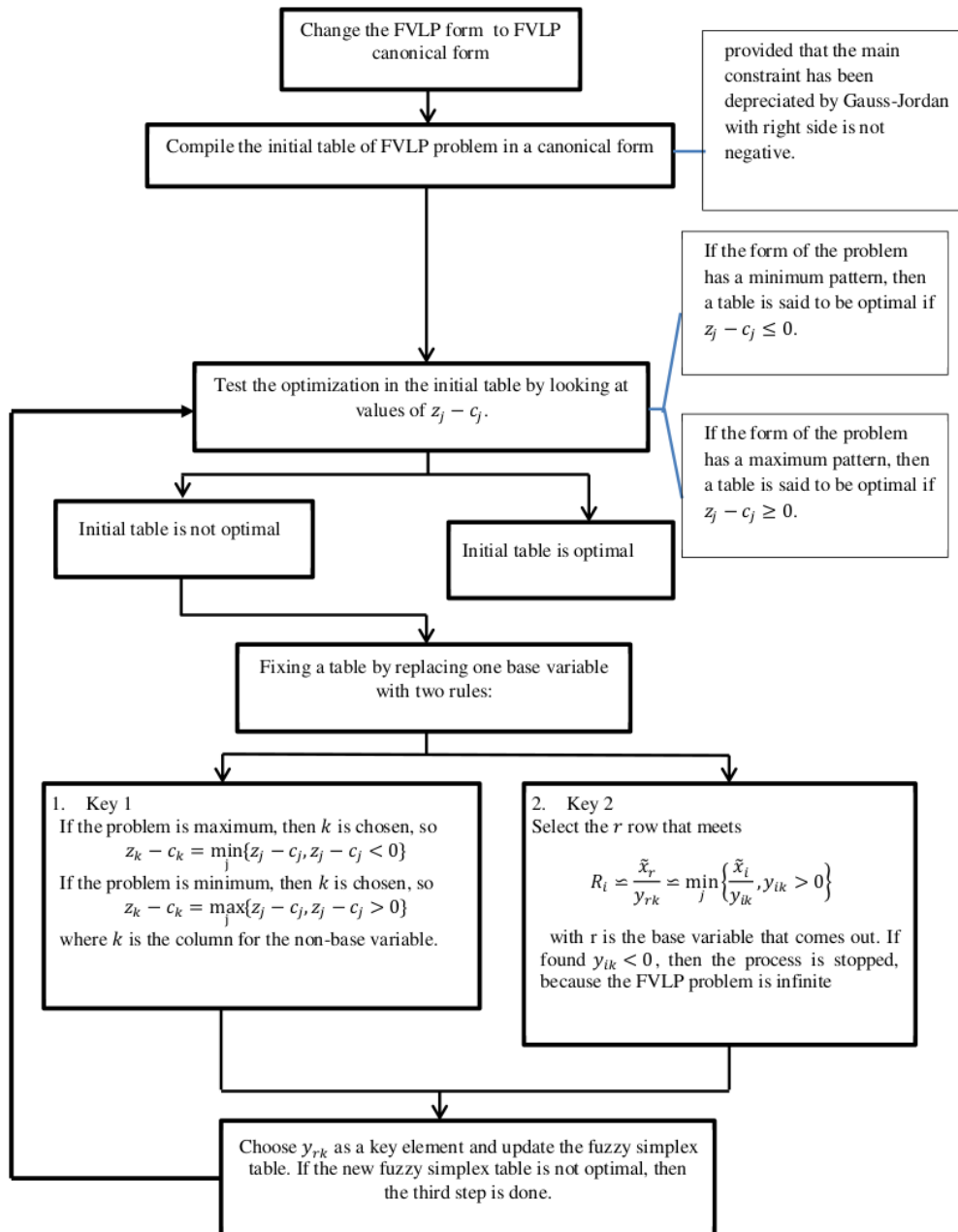
Change the FVLP form to FVLP canonical form

provided that the main constraint has been depreciated by Gauss-Jordan with right side is not negative.

Compile the initial table of FVLP problem in a canonical form

If the form of the problem has a minimum pattern, then a table is said to be optimal if $z_j - c_j \leq 0$.

Test the optimization in the initial table by looking at values of $z_j - c_j$.

If the form of the problem has a maximum pattern, then a table is said to be optimal if $z_j - c_j \geq 0$.

Initial table is not optimal

Initial table is optimal

Fixing a table by replacing one base variable with two rules:

1. Key 1
If the problem is maximum, then k is chosen, so
$$z_k - c_k = \min_j \{z_j - c_j, z_j - c_j < 0\}$$
If the problem is minimum, then k is chosen, so
$$z_k - c_k = \max_j \{z_j - c_j, z_j - c_j > 0\}$$
where k is the column for the non-base variable.

2. Key 2
Select the r row that meets
$$R_i = \frac{\tilde{x}_r}{y_{rk}} = \min_j \left\{\frac{\tilde{x}_i}{y_{ik}}, y_{ik} > 0\right\}$$
with r is the base variable that comes out. If found $y_{ik} < 0$, then the process is stopped, because the FVLP problem is infinite

Choose y_{rk} as a key element and update the fuzzy simplex table. If the new fuzzy simplex table is not optimal, then the third step is done.

Figure 2. Steps for solving FVLP with simpleks fuzzy method.

The form of mathematics from the Linear Program Case that has been given a fuzzy value on the right-hand coefficient of the objective function is:

Objective function

Max $\tilde{Z} = \sum_{j=1}^{n} c_j \tilde{x}_j$

Subject to $\sum_{j=1}^{n} a_{ij} \tilde{x}_j \lesssim \tilde{b}_i$ $\qquad (i = 1,2,...,m)$

The mathematical model above is a form of FVLP (Fuzzy Variable Linear Programming). FVLP is a flexible mathematical technique, which optimizes the use of limited resources to achieve an optimal goal, in this case maximizing the waste load. The problem of this linear fuzzy programming model will be solved using the fuzzy simplex method.

To form a mathematical model for wastewater at the Sewon's WWTP, Bantul, Yogyakarta, it is first to know the scheme of wastewater treatment systems in the stabilization pond which can be seen in Figure 3.
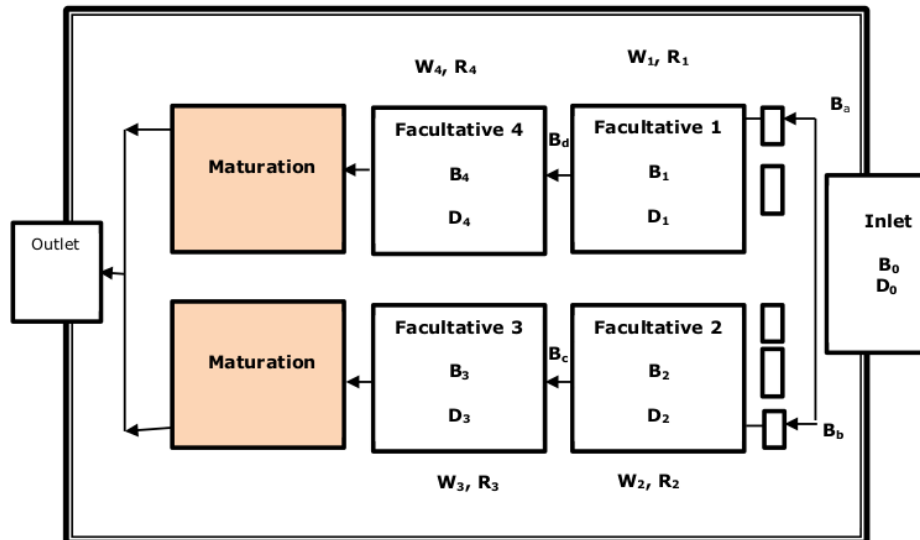
Figure 3. Wastewater Treatment Scheme

with:

B_0 : Initial waste load in the inlet (kg / day)
B_i : Number of wastewater treatment in the pond i (kg /day)
D_0 : Debit of wastewater in the inlet (m³ / day)
D_i : Debit of wastewater in the pond i (m³ / day)
B_a : wastewater load before entering the facultative pond 1 (kg / day)
B_b : wastewater load before entering the facultative pond 2 (kg / day)
B_c : wastewater load before entering the facultative pond 3 (kg / day)
B_d : wastewater load before entering the facultative pond 4 (kg / day)
W_i : BOD solube pond i
R_i : BOD particule pond i
SK : Water quality standards

The total waste load treated is the amount of waste load processed in facultative ponds 1 (B_1), facultative 2 (B_2), facultative 3 (B_3), and facultative 4 (B_4) . In this discussion, it is assumed that wastewater treatment systems occur in facultative ponds and ignore maturation ponds. Formulation of constraints to pay attention to BOD parameters so that they meet water quality standards. In this discussion, the amount of wastewater treatment in pond 1 (B_1), not greater than the load of wastewater before entering the facultative pond 1 (B_a), (B_2) no greater than B_b, B_3 no greater than B_c , and B_4 is not greater than B_d.

Based on Figure 3, a linear program model can be formed with the following variables:

Objective function

$$\text{Max } \tilde{Z} = \tilde{B}_1 + \tilde{B}_2 + \tilde{B}_3 + \tilde{B}_4$$

Subject to

$$
\begin{aligned}
W_1 + R_1 &\lesssim \widetilde{SK} \\
W_2 + R_2 &\lesssim \widetilde{SK} \\
W_3 + R_3 &\lesssim \widetilde{SK} \\
W_4 + R_4 &\lesssim \widetilde{SK} \\
B_1 &\lesssim B_a \\
B_2 &\lesssim B_b \\
B_1 + B_4 &\lesssim B_a \\
B_2 + B_3 &\lesssim B_a
\end{aligned}
\qquad (4)
$$

The BOD solube (W_i) can be calculated by the equation [6,7]:

$$W_i = \frac{L_i}{1 + K_{1T}t}$$

with:

L_i : BOD influen pond i (mg/L)

K_{1T} : first order constant, where $K_{1T} = K_{1(20)}(1.05)^{T-20}$, $K_{1(20)} = 0.3$ per day for primary ponds and 0.1 per day for secondary ponds

T : temperature

t_f : water retention time

From the description, equation (4) can be written as:

Objective function

$$\text{Max } \tilde{Z} = \tilde{B}_1 + \tilde{B}_2 + \tilde{B}_3 + \tilde{B}_4$$

Subject to

$$
\begin{aligned}
\frac{L_1}{1 + K_{1T}t_f} + R_1 &\lesssim \widetilde{SK} \\[4pt]
\frac{L_2}{1 + K_{1T}t_f} + R_2 &\lesssim \widetilde{SK} \\[4pt]
\frac{L_3}{1 + K_{1T}t_f} + R_3 &\lesssim \widetilde{SK} \\[4pt]
\frac{L_4}{1 + K_{1T}t_f} + R_4 &\lesssim \widetilde{SK} \\[4pt]
B_1 &\lesssim B_a \\
B_2 &\lesssim B_b \\
B_1 + B_4 &\lesssim B_a \\
B_2 + B_3 &\lesssim B_a
\end{aligned}
\qquad (5)
$$

Equation (5) can be described again by substituting L_i : BOD influent pond i, as a waste load before entering the pond (B_a, B_b, B_c, B_d). So that the fuzzy linear programming model become:

Objective function

$$\text{Max } \tilde{Z} = \tilde{B}_1 + \tilde{B}_2 + \tilde{B}_3 + \tilde{B}_4$$

Subject to

$$\frac{B_a}{1 + K_{1T}t_f} + R_1 \lesssim \widetilde{SK}$$

$$\frac{B_b}{1 + K_{1T}t_f} + R_2 \lesssim \widetilde{SK}$$

$$\frac{B_c}{1 + K_{1T}t_f} + R_3 \lesssim \widetilde{SK}$$

$$\frac{B_d}{1 + K_{1T}t_f} + R_4 \lesssim \widetilde{SK}$$

$$B_1 \lesssim B_a$$
$$B_2 \lesssim B_b$$
$$B_1 + B_4 \lesssim B_a$$
$$B_2 + B_3 \lesssim B_a \qquad\qquad\qquad (6)$$

The above mathematical model can be completed, assuming the following:

1. The waste load before entering pond 1 (B_a) and 2 (B_b) is half of the waste load on the inlet [9]. The waste load at the inlet (B_0) is the debit multiplied by the BOD concentration in the inlet. In this discussion, the waste load on the inlet can be seen from the number of population multiplied by the BOD concentration in the inlet. Sewon's WWTP domestic wastewater management services cover 3 City and Regency areas in D.I. Yogyakarta, that are Sleman Regency with 795 connections, Bantul Regency with 730 connections, and Yogyakarta City with 12,804 connections. So that the total service is 14,329 house connections. The large number of population, BOD load can be seen in table 1.

Table 1. Total Population, mass BOD per person, BOD Inlet, B_a , B_b

Total Population	119,900 person
BOD per person	46 gr/ person/ day
Massa BOD Inlet	5,515.4 kg/day
B_a	2,757.7 kg/day
B_b	2,757.7 kg/day

2. The first order constant (K_{1T}) can be calculated by the formula $K_{1T} = K_{1(20)}(1.05)^{T-20}$, $K_{1(20)} = 0.3$ per day for the primary ponds and 0.1 per day for the secondary ponds. The average temperature is taken by 25°C, so it is obtained:

$$K_{1T} = K_{1(20)}(1.05)^{T-20}$$
$$= 0.3 (1.05)^{25-20}$$
$$= 0.38288$$

3. Water Retention time (t_f) in facultative pond can be obtained by looking at the average retention time in the January-December 2016 period, can be seen in table 2. Average of water retention time is 4.83 day

4. Domestic Wastewater Quality Standards refer to DIY Governor Decree number 214 / KPTS / 1991. The standard of wastewater quality for water bodies in Group B is in Group II, the BOD parameter is 50 mg / L. The Yogyakarta City Government has also regulated the Management of

Domestic Wastewater through the Yogyakarta City Regulation No. 6 of 2009, the quality standard of domestic wastewater with the maximum level of BOD parameters is 100 mg / L. From the explanation above, water quality standards will be used as constraints of the right-hand coefficient in the form of fuzzy numbers, is $\widetilde{SK} = [50,67,83,100]$.

Table 2. Waste Retention Time from January-December 2016 in Facultative Pond

Month	Retention Time (day)
January	3.97
February	3.68
March	3.94
April	4.13
May	4.31
June	4.82
July	4.11
August	5.6
September	5.8
October	5.43
November	6.3
December	5.83
Average	4.83

3. Results and discussion
Based on the assumptions, the linear fuzzy program model can be described by substituting the known variable.
Objective function
Max $\widetilde{Z} = \widetilde{B}_1 + \widetilde{B}_2 + \widetilde{B}_3 + \widetilde{B}_4$
Subject to

$$\frac{B_a}{1 + 0.38288\,(4.83)} + R_1 \lesssim [50,67,83,100]$$

$$\frac{B_b}{1 + 0.38288\,(4.83)} + R_2 \lesssim [50,67,83,100]$$

$$\frac{B_c}{1 + 0.38288\,(4.83)} + R_3 \lesssim [50,67,83,100]$$

$$\frac{B_d}{1 + 0.38288\,(4.83)} + R_4 \lesssim [50,67,83,100]$$

$$B_1 \lesssim B_a$$
$$B_2 \lesssim B_b$$
$$B_1 + B_4 \lesssim B_a$$
$$B_2 + B_3 \lesssim B_a \tag{7}$$

The equation system is solved by using the fuzzy simplex method. From calculations, fuzzy linear programming can degrade BOD and can meet water quality standards with wastewater retention time of 4.83 days.

4. Conclusion
Fuzzy linear programming can be used to solve wastewater treatment problems. This study determines the optimal waste load and degradation rate of organic matter, and the results meet water quality standards.

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