ABSTRACT

DYNAMIC MODELLING ANALYSIS OF TUBERCULOSIS TRANSMISSION WITH REINFECTION FACTORS AND OPTIMAL CONTROL STRATEGIES

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This study discusses a mathematical model of Tuberculosis transmission by dividing the population into five subpopulations: susceptible, vaccinated, exposed, actively infected, and recovered. The model is modified by adding parameters for the direct infection rate, the rate at which vaccinated individuals become immune, and the treatment rate for exposed individuals. The model system yields two equilibrium points, namely the disease-free equilibrium and the endemic equilibrium. The local and global stability of these equilibria are analyzed using the Routh–Hurwitz stability criterion and Lyapunov functions. The disease-free equilibrium is locally asymptotically stable and globally stable when the basic reproduction number is less than one, whereas the endemic equilibrium is locally asymptotically stable and globally stable when the basic reproduction number is greater than one. Sensitivity analysis further shows that the recruitment rate and the disease transmission rate have the greatest influence on increasing the basic reproduction number, while the recovery rate due to treatment is effective in reducing it. Furthermore, this study applies optimal control to a case study of Central Java Province to determine optimal strategies for reducing Tuberculosis transmission through prevention education, vaccination, and treatment. The optimal control problem is solved using Pontryagin's Minimum Principle. Numerical simulations of the optimal control system are performed iteratively using the Forward–Backward Sweep method, in which the state equations are solved forward in time and the adjoint equations are solved backward in time, to minimize the number of exposed and actively infected individuals. Among the four strategies considered, the combination of all three controls in Strategy IV produces the most optimal results, achieving reductions of 99.97% in both exposed and actively infected individuals; therefore, this strategy is considered the most effective in suppressing the spread of Tuberculosis.

*Keywords**:** Optimal control, Lyapunov function, Pontryagin's Minimum Principle, Tuberculosis.*